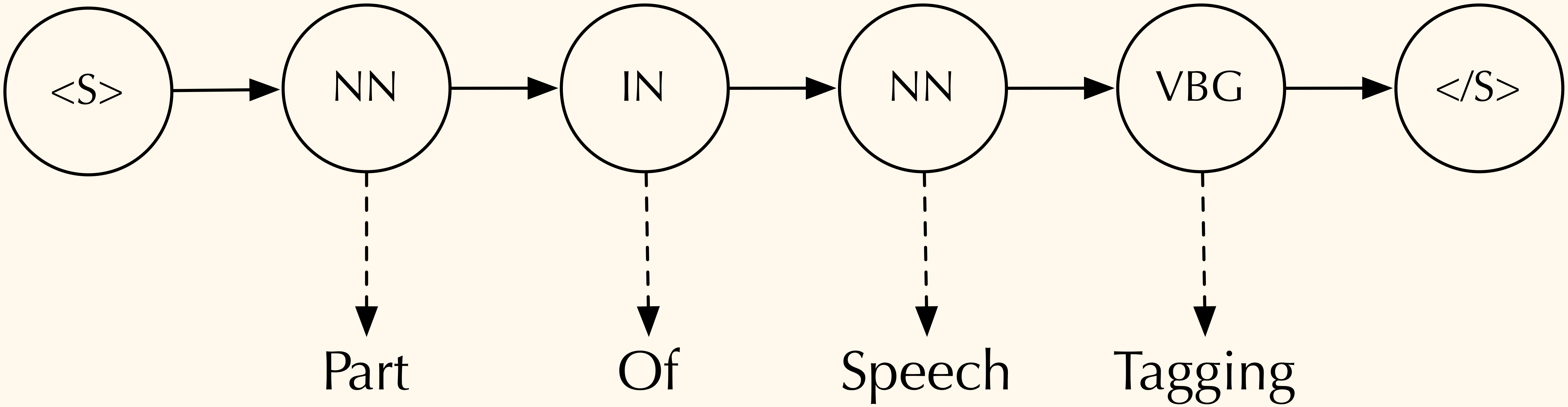

Discrete HMMs, part 2: Viterbi!

Steven Bedrick
CS/EE 5/655, 10/27/14

Quick Review:

Stochastic PoS techniques rely entirely on probability.

The goal of a stochastic PoS tagger is to find:

$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n | w_1^n)$$

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Via Bayes' Rule:

$$\hat{t}_1^n = \arg\max_{t_1^n} \underbrace{P(w_1^n | t_1^n)}_{\text{Likelihood}} \underbrace{P(t_1^n)}_{\text{Prior}}$$

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After making a few simplifying assumptions:

$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n | w_1^n)$$

Via Bayes' Rule:

$$\hat{t}_1^n = \arg\max_{t_1^n} P(w_1^n | t_1^n) P(t_1^n)$$

Likelihood Prior

After making a few simplifying assumptions:

$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n | w_1^n) \approx \arg\max_{t_1^n} \prod_{i=1}^{n} P(w_i | t_i) P(t_i | t_{i-1})$$

$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n | w_1^n)$$

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Probability of
word given tag

$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n|w_1^n)$$

Via Bayes' Rule:

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Likelihood Prior

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$$\hat{t}_1^n = \arg\max_{t_1^n} P(t_1^n|w_1^n) \approx \arg\max_{t_1^n} \prod_{i=1}^{n} P(w_i|t_i)P(t_i|t_{i-1})$$

Probability of word given tag

Probability of tag given previous tag

скрытая марковская модель

Andrei Andreievich Markov
1856–1922

HMMs are a type of stochastic model used to examine sequential data.

The basic idea: there are two parameters changing over time, but we can only directly observe one of them. We want to know about the other.

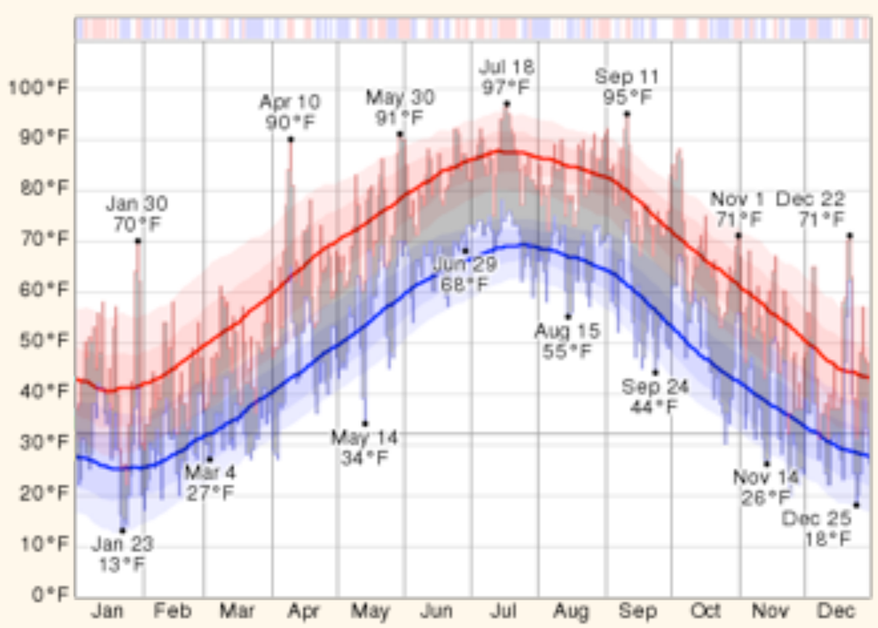
Q: { *Hot, Cold* }

A:

	Hot	Cold
Hot	0.7	0.3
Cold	0.6	0.4

B:

	1	2	3
Hot	0.2	0.4	0.4
Cold	0.5	0.4	0.1



$a_{0,Hot/Cold}$:

	Start
Hot	0.8
Cold	0.2

Note: for this demonstration, we are ignoring a_F.

There are three fundamental kinds of questions that we can ask with an HMM:

1. *Likelihood:* Given a sequence of states, what is the most likely observed sequence? *or* How likely is a given observation sequence?

2. *Decoding:* Given an observation sequence and a fully-specified HMM, what is the most likely sequence of states to have produced that observation?

3. *Learning:* Given an observation sequence and a set of states, what are the likely transition and emission probabilities (A and B)?

There are three fundamental kinds of questions that we can ask with an HMM:

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3. *Learning:* Given an observation sequence and a set of states, what are the likely transition and emission probabilities (A and B)?

Let's say we have a sequence of diary entries:

$$O = 3, 1, 3$$

How likely is this sequence given the model described earlier? $P(O|\lambda)$

The *forward algorithm* uses dynamic programming to calculate this efficiently…

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… by building a trellis α such that each $\alpha_t(j)$ represents:

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the probability of the machine being in state *j*,

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the probability of the machine being in state *j*,

... given the first t observations ("forward probability").

The *forward algorithm* uses dynamic programming to calculate this efficiently...

... by building a trellis α such that each $\alpha_t(j)$ represents:

the probability of the machine being in state *j*,

... given the first t observations ("forward probability").

Formally: $\alpha_t(j) = P(o_1, o_2 ... o_t, q_t = j | \lambda)$

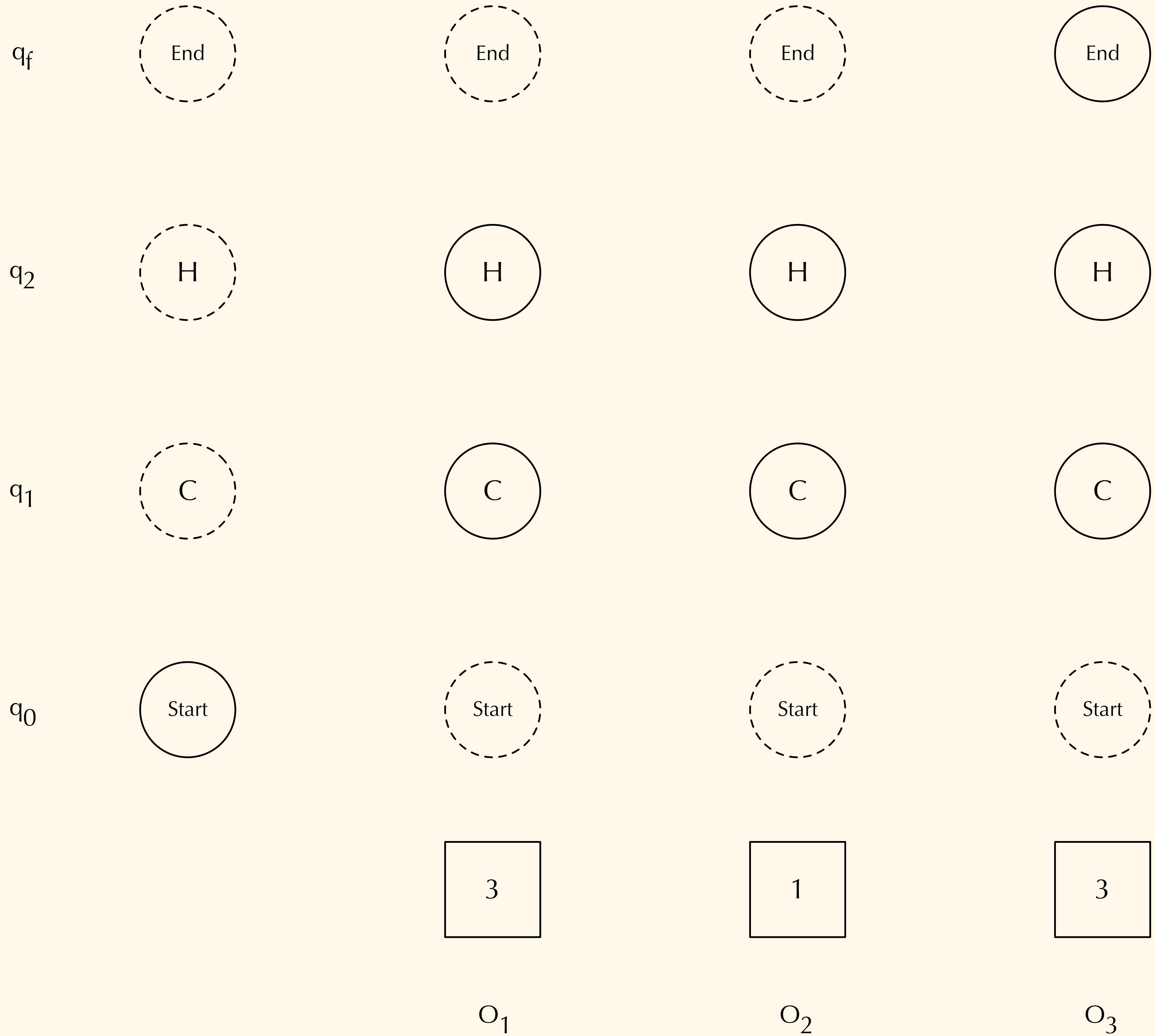
Calculating $\alpha_t(j) = P(o_1, o_2 ... o_t, q_t = j | \lambda)$ is fairly straightforward:

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

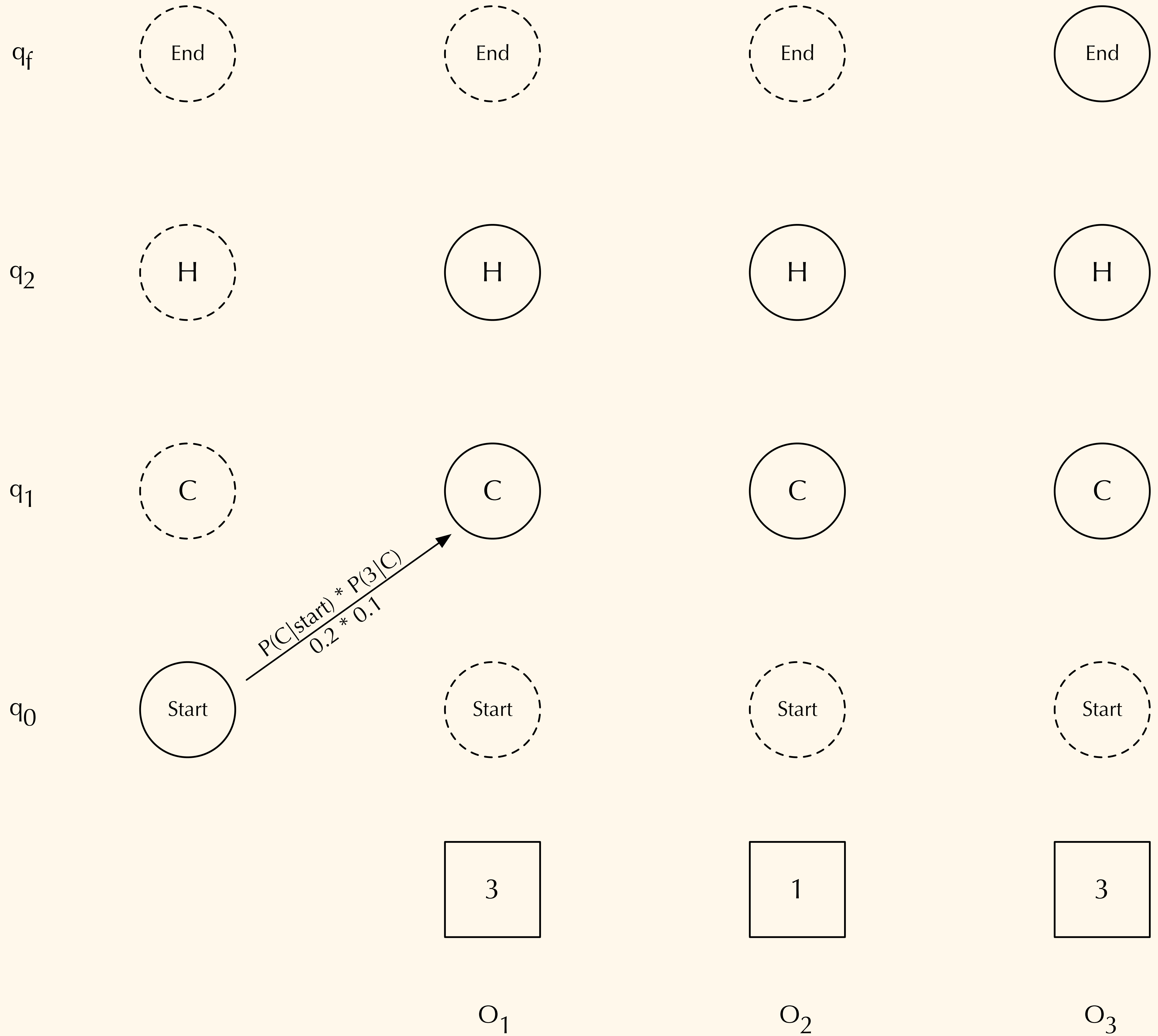
Previous time step's forward probability for state *i*

Emission likelihood for symbol o_t given current state *j*

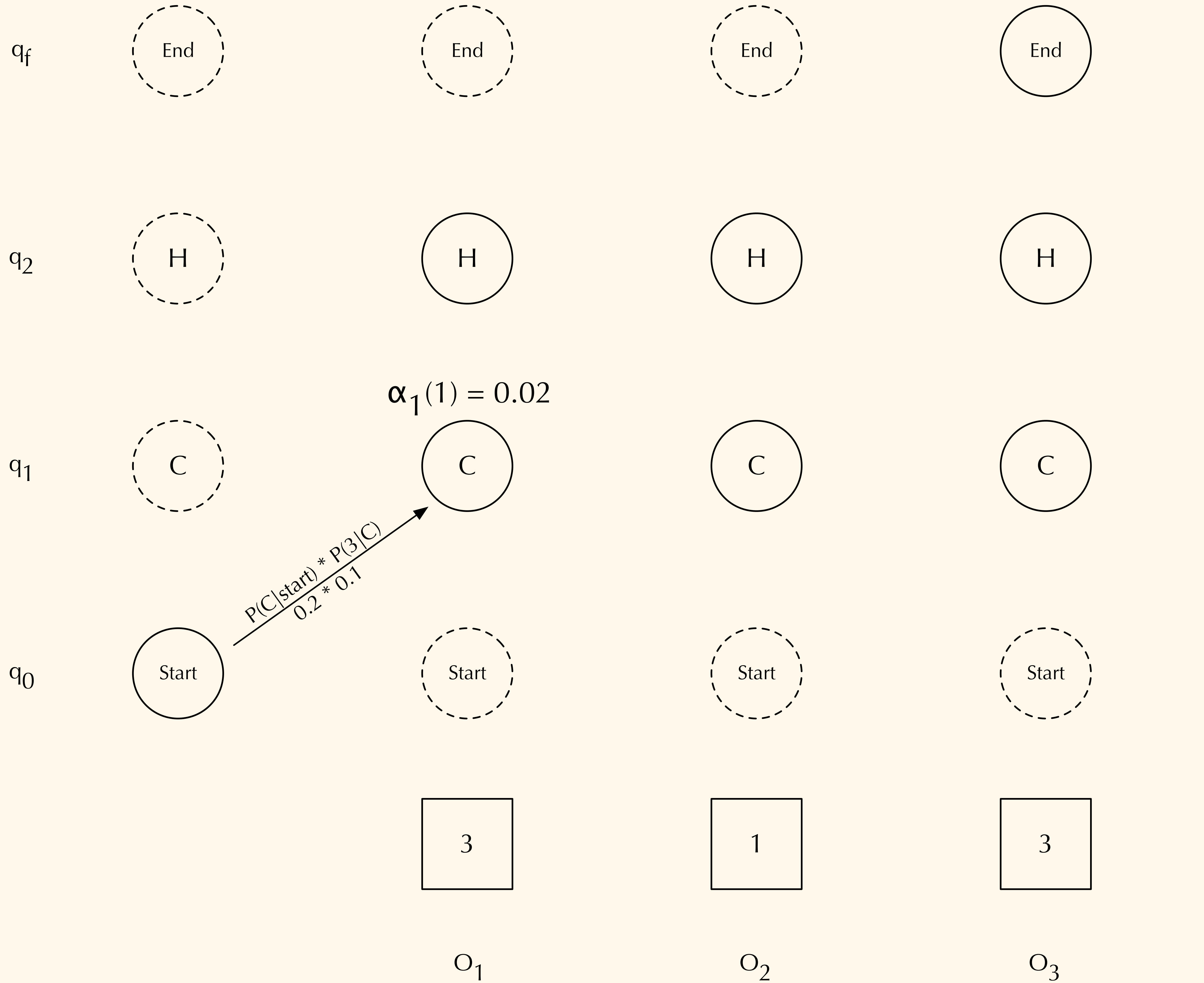
Transition prob. from previous state *i* to current state *j*



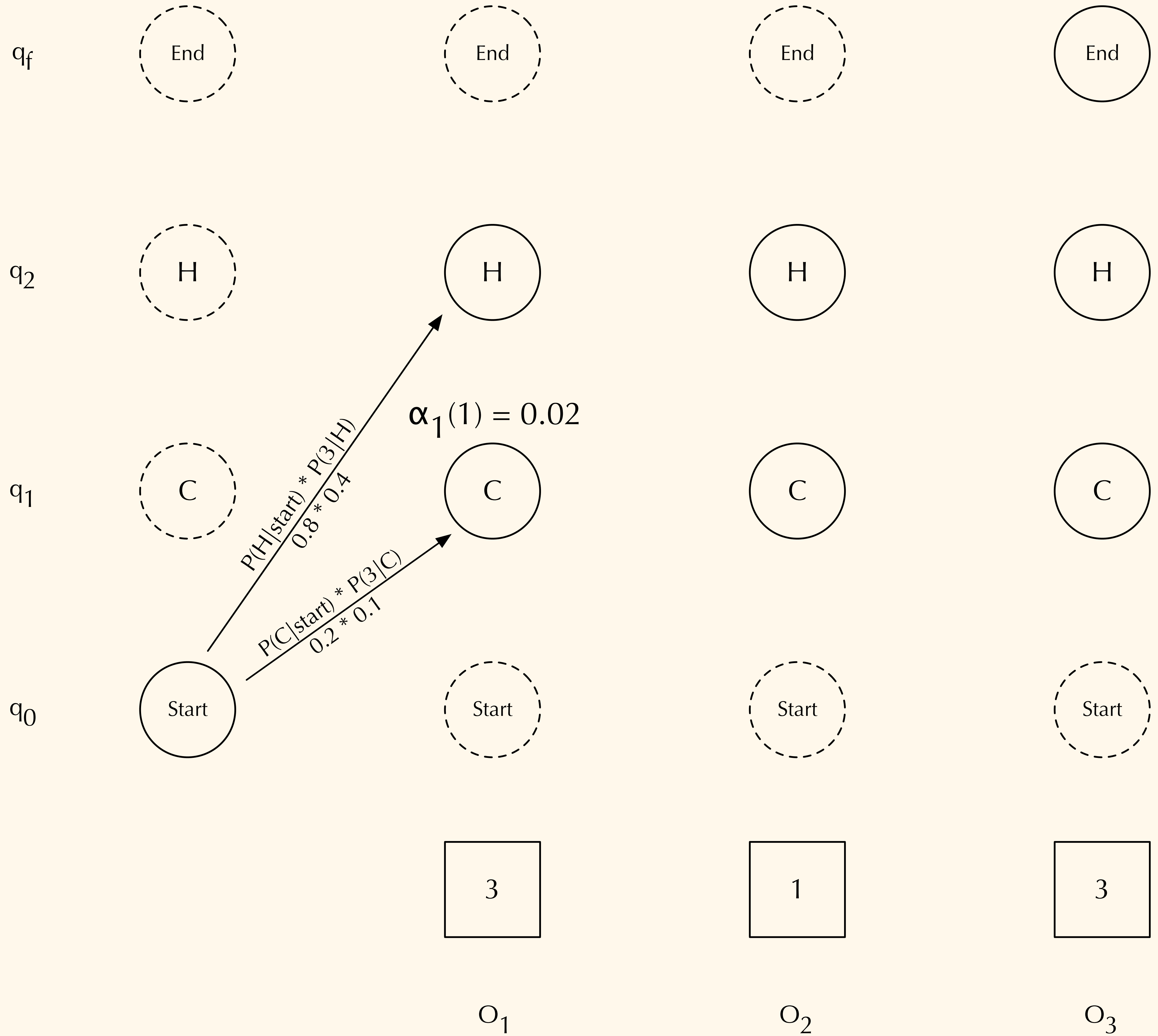
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



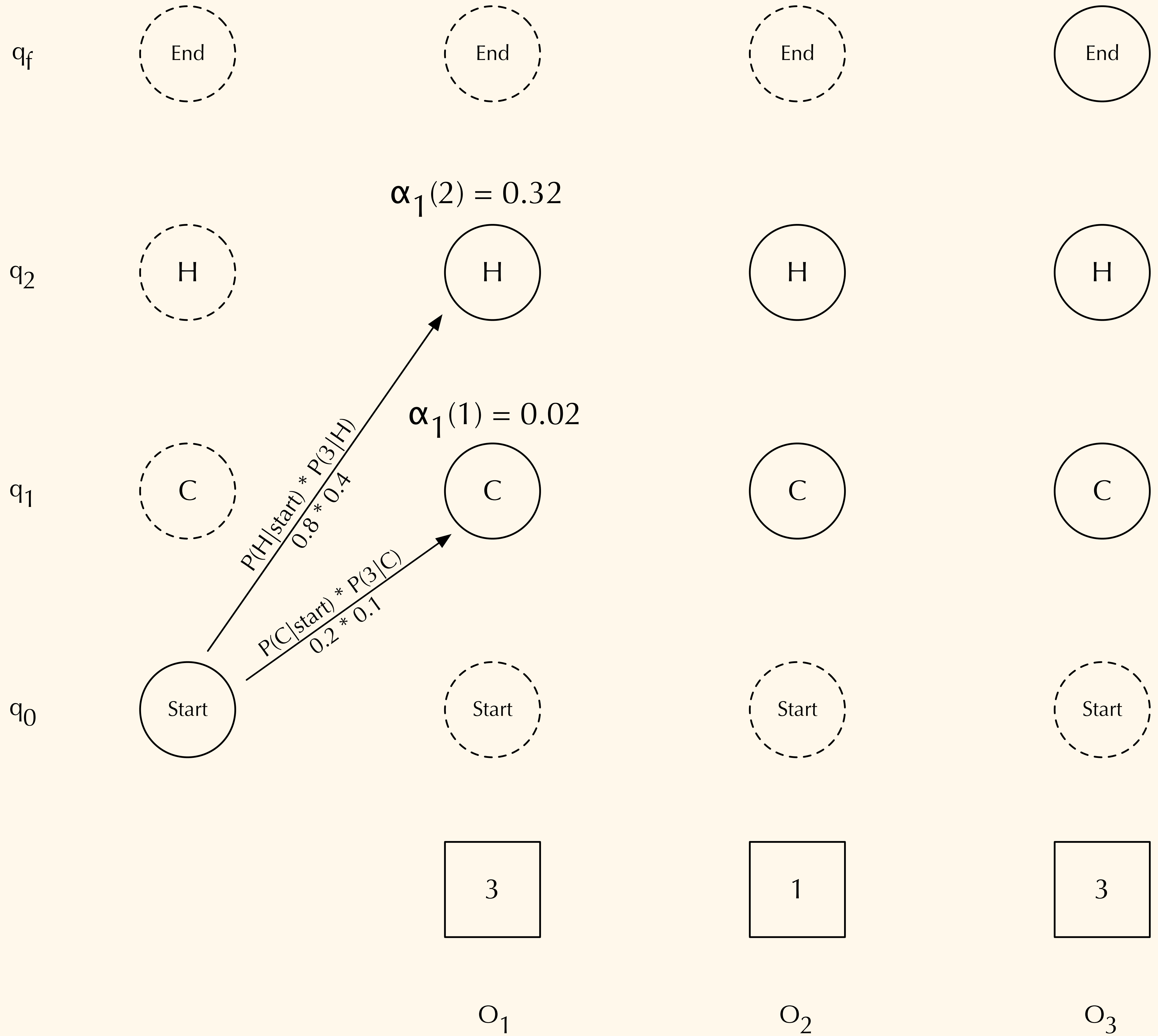
qf
End
End
End
End
q2
H
H
H
H
q1
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
q0
Start
Start
Start
Start
3
1
3
O1
O2
O3



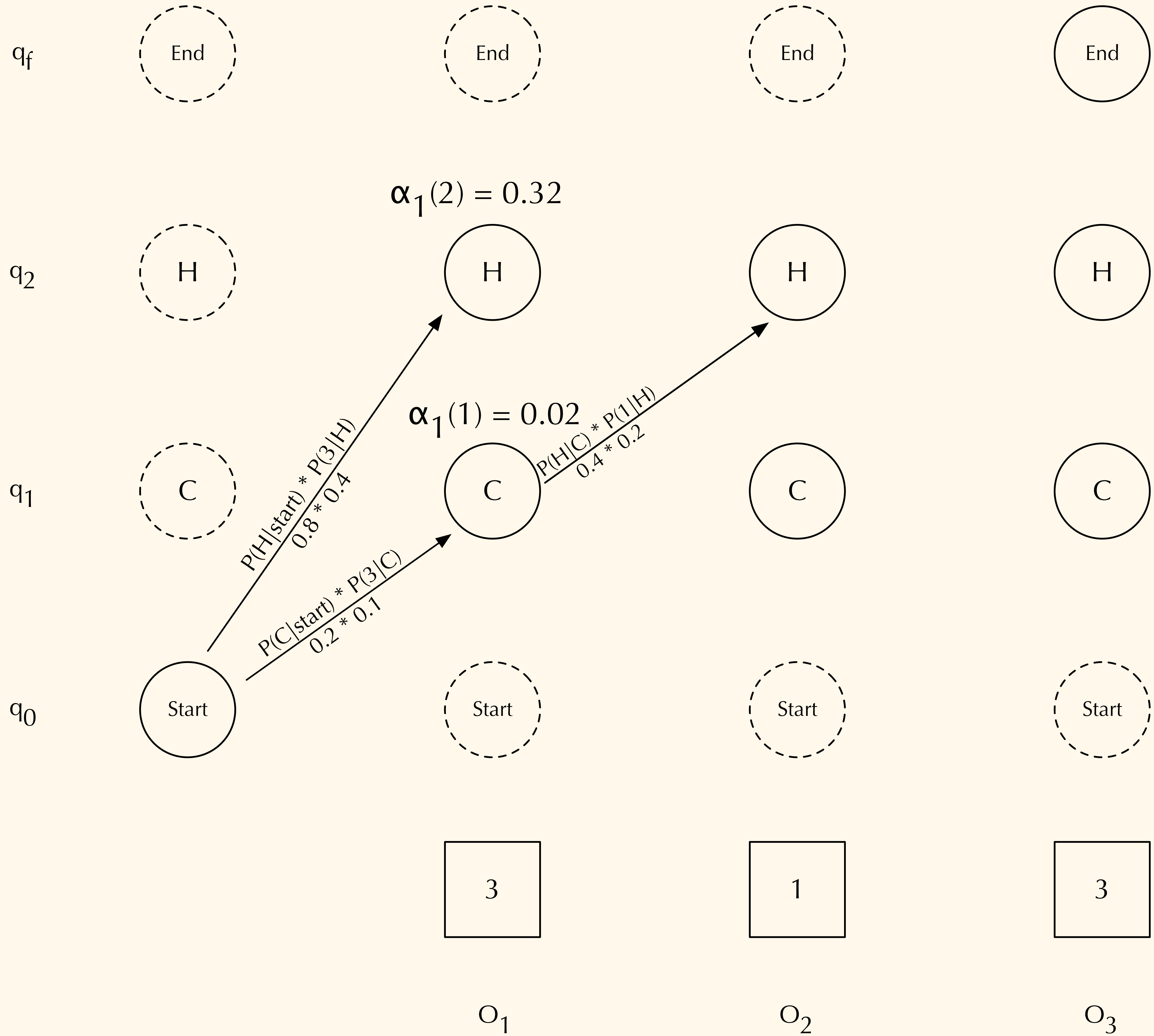
q_f
q_2
q_1
q_0
End
H
C
Start
α_1(1) = 0.02
P(C|start) * P(3|C)
0.2 * 0.1
3
1
3
O_1
O_2
O_3



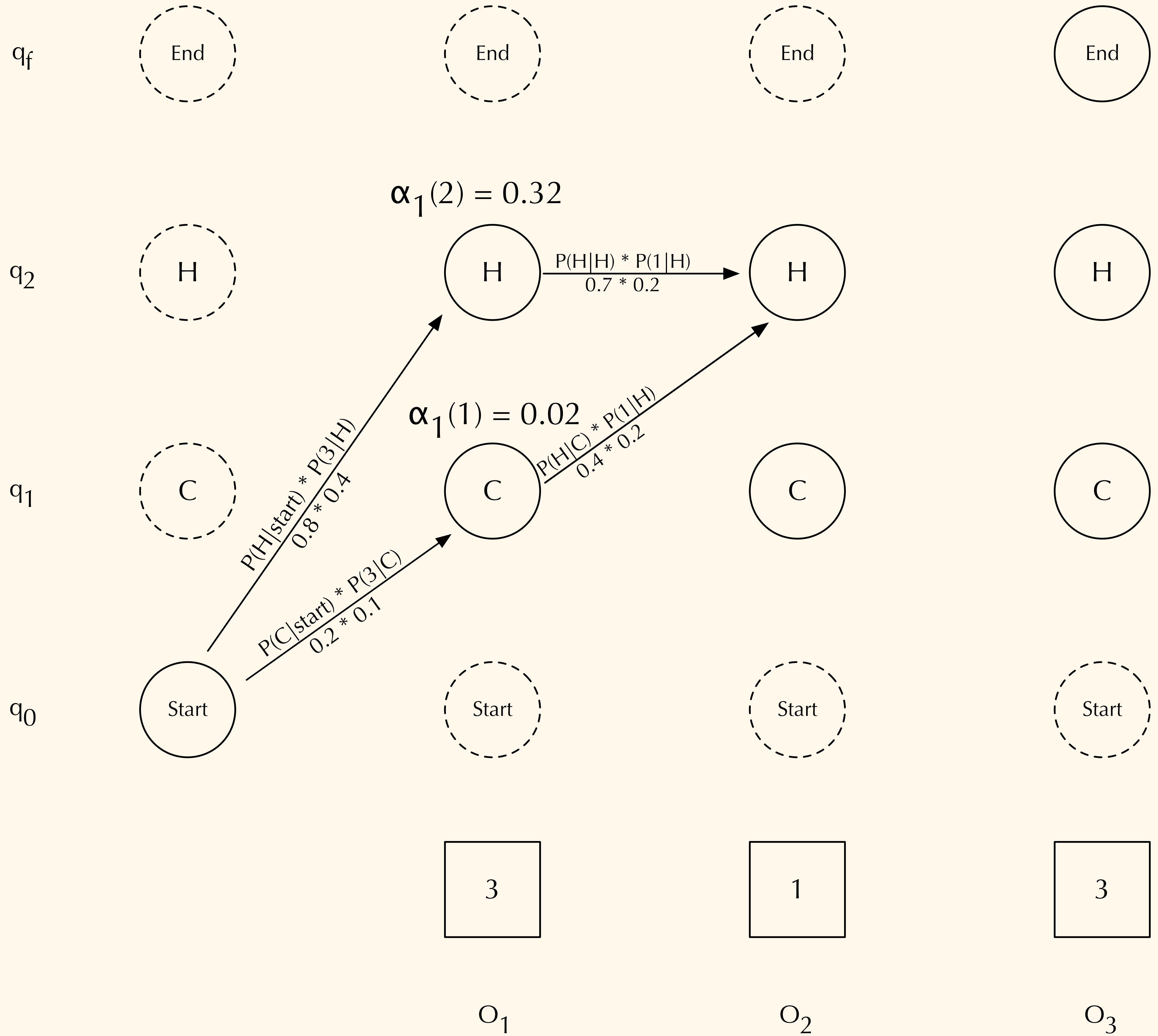
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
$\alpha_1(1) = 0.02$
3
1
3
o_1
o_2
o_3



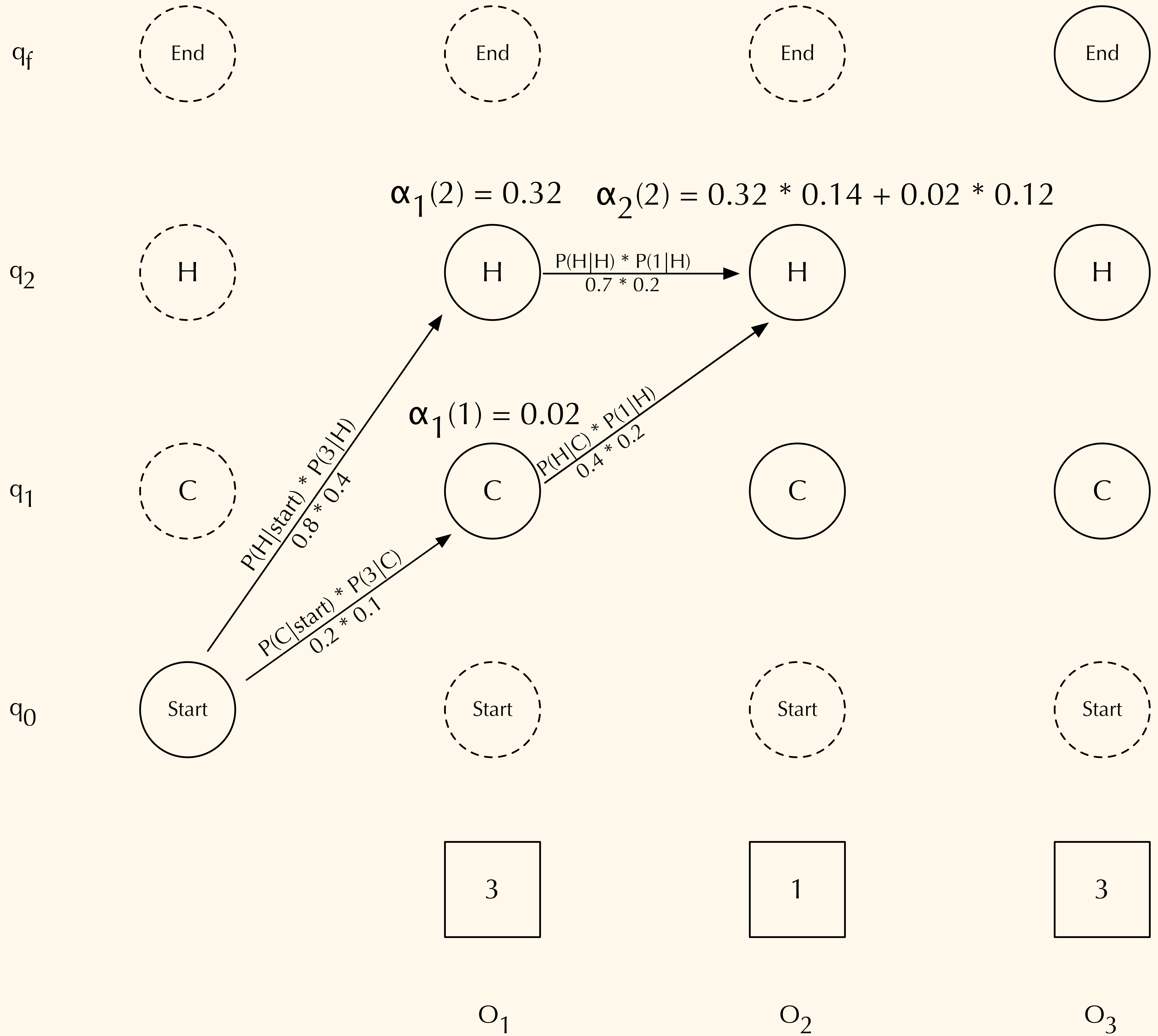
q_f
q_2
q_1
q_0
End
End
End
End
α_1(2) = 0.32
H
H
H
H
α_1(1) = 0.02
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3

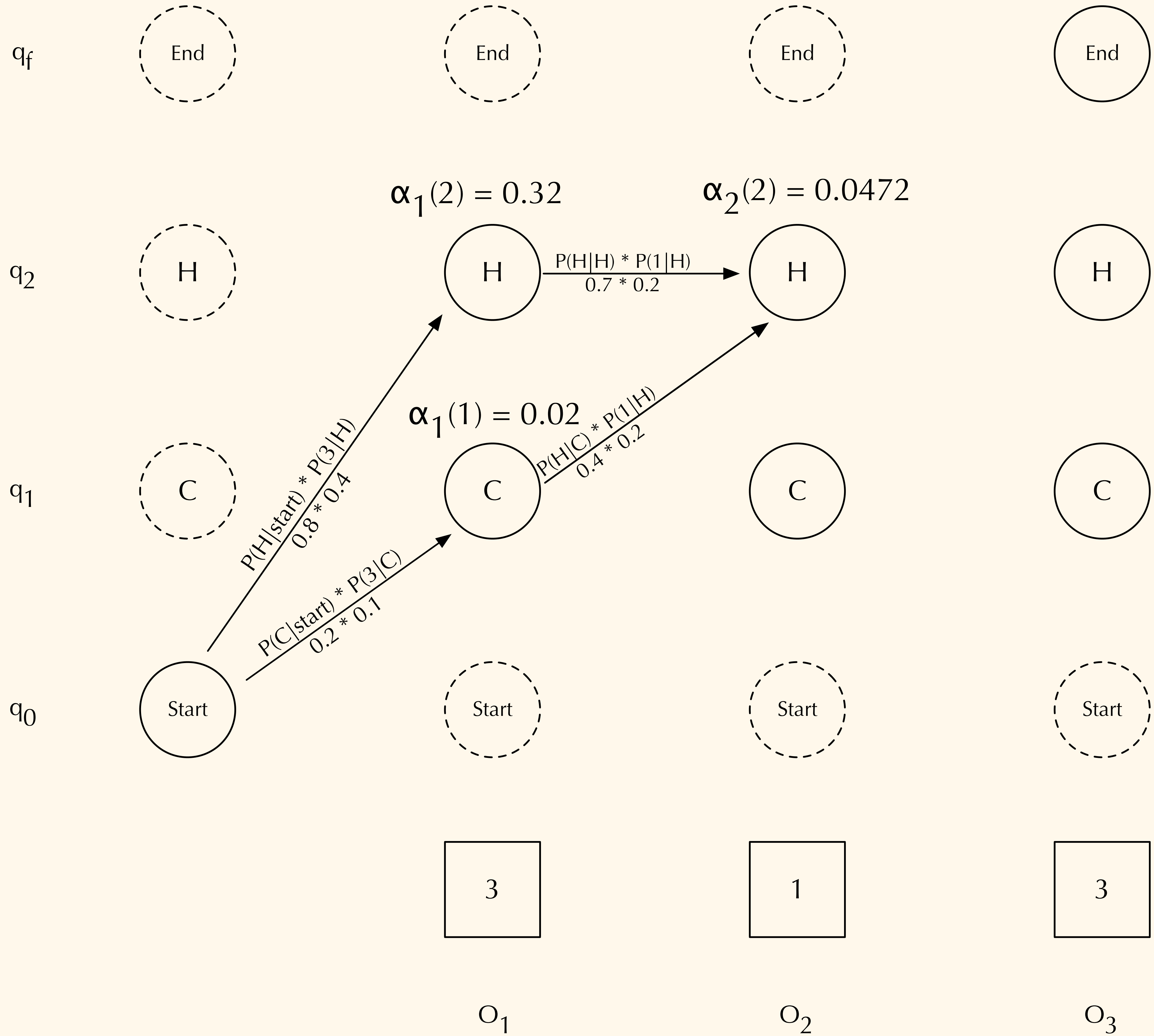


q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
H
H
H
H
$\alpha_1(1) = 0.02$
C
C
C
C
Start
Start
Start
Start
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
P(H|C) * P(1|H)
0.4 * 0.2
3
1
3
O_1
O_2
O_3

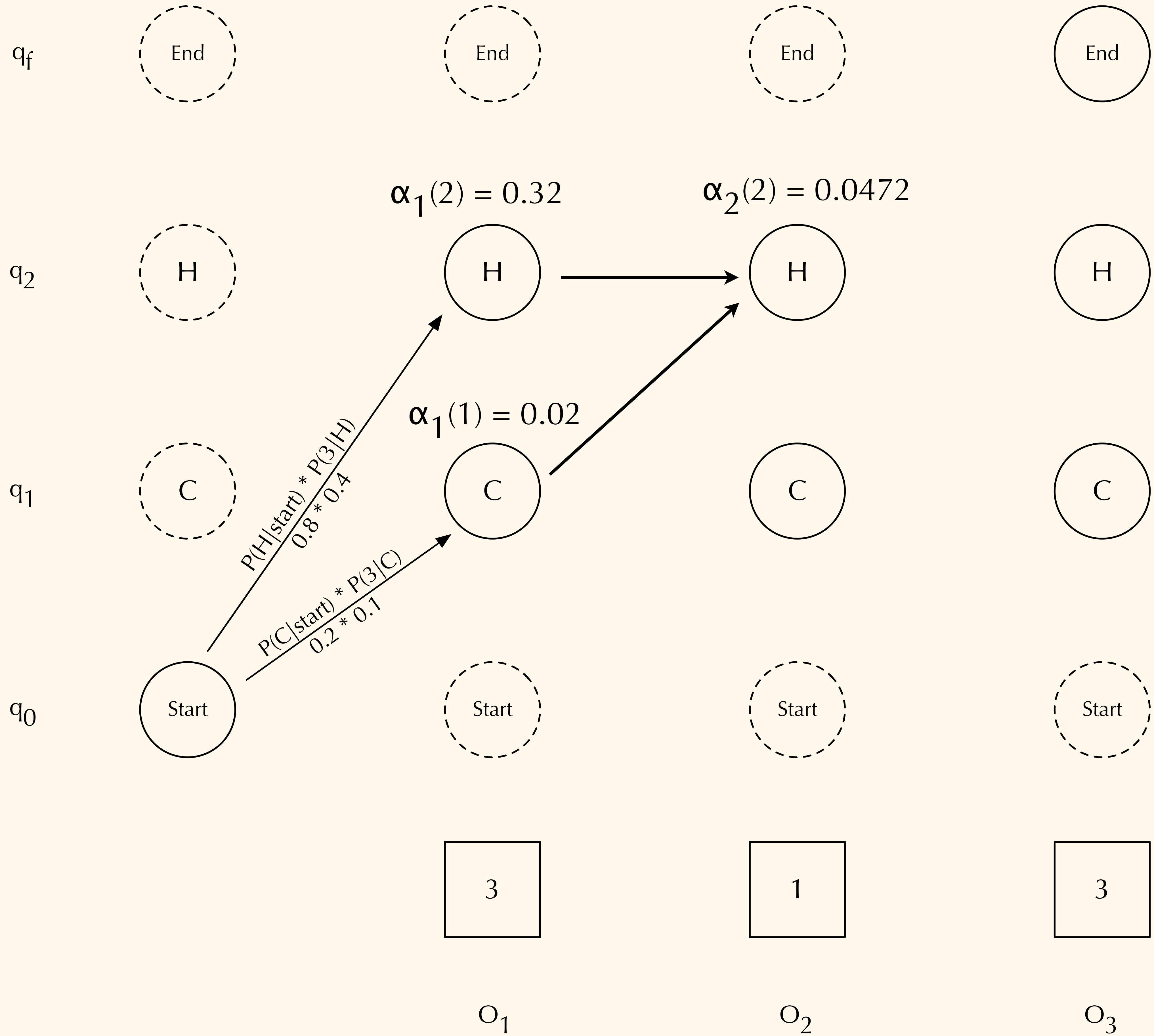


q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$\alpha_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
P(H|C) * P(1|H)
0.4 * 0.2
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3

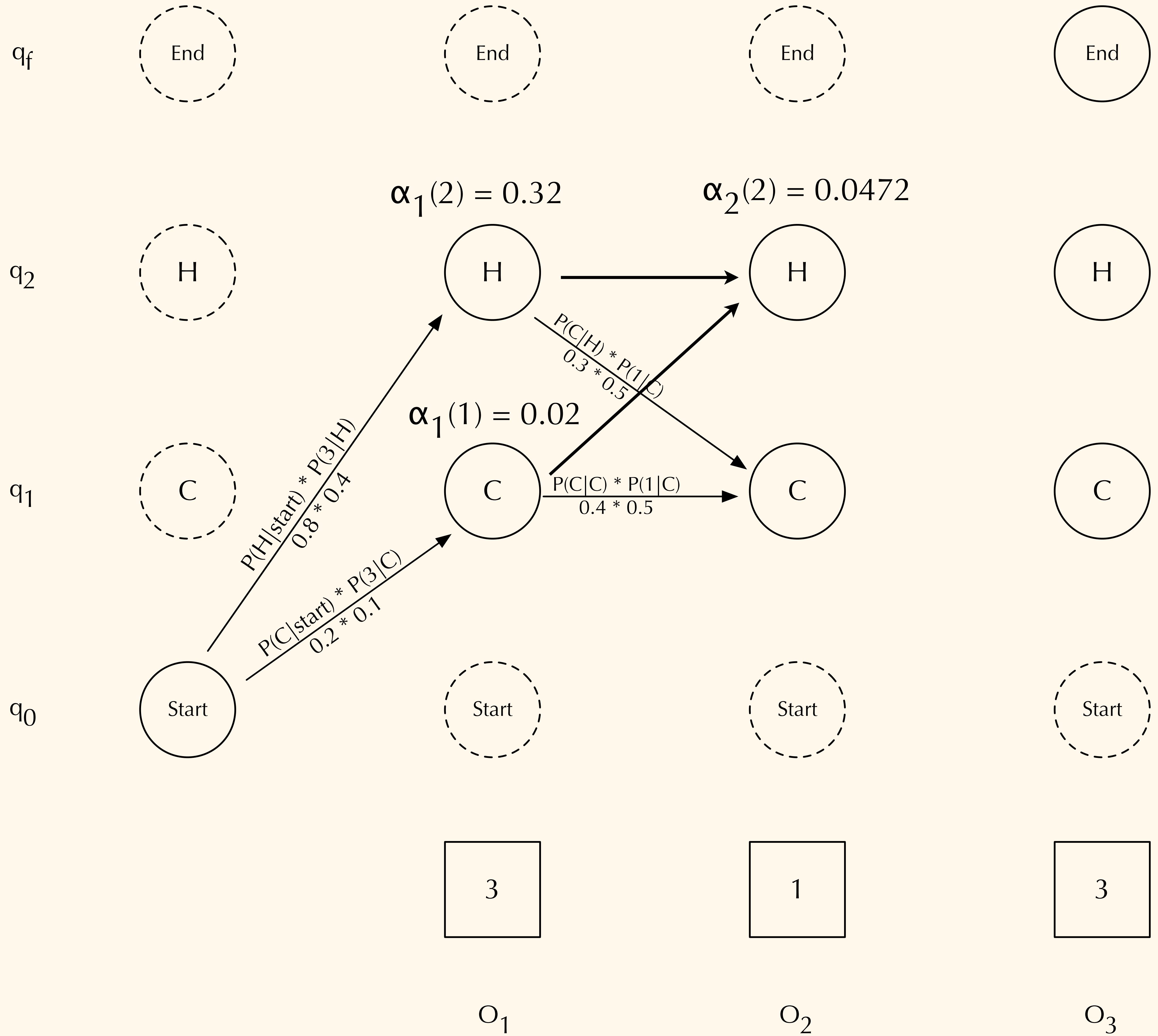

q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.32 * 0.14 + 0.02 * 0.12$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$\alpha_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(H|C) * P(1|H)
0.4 * 0.2
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



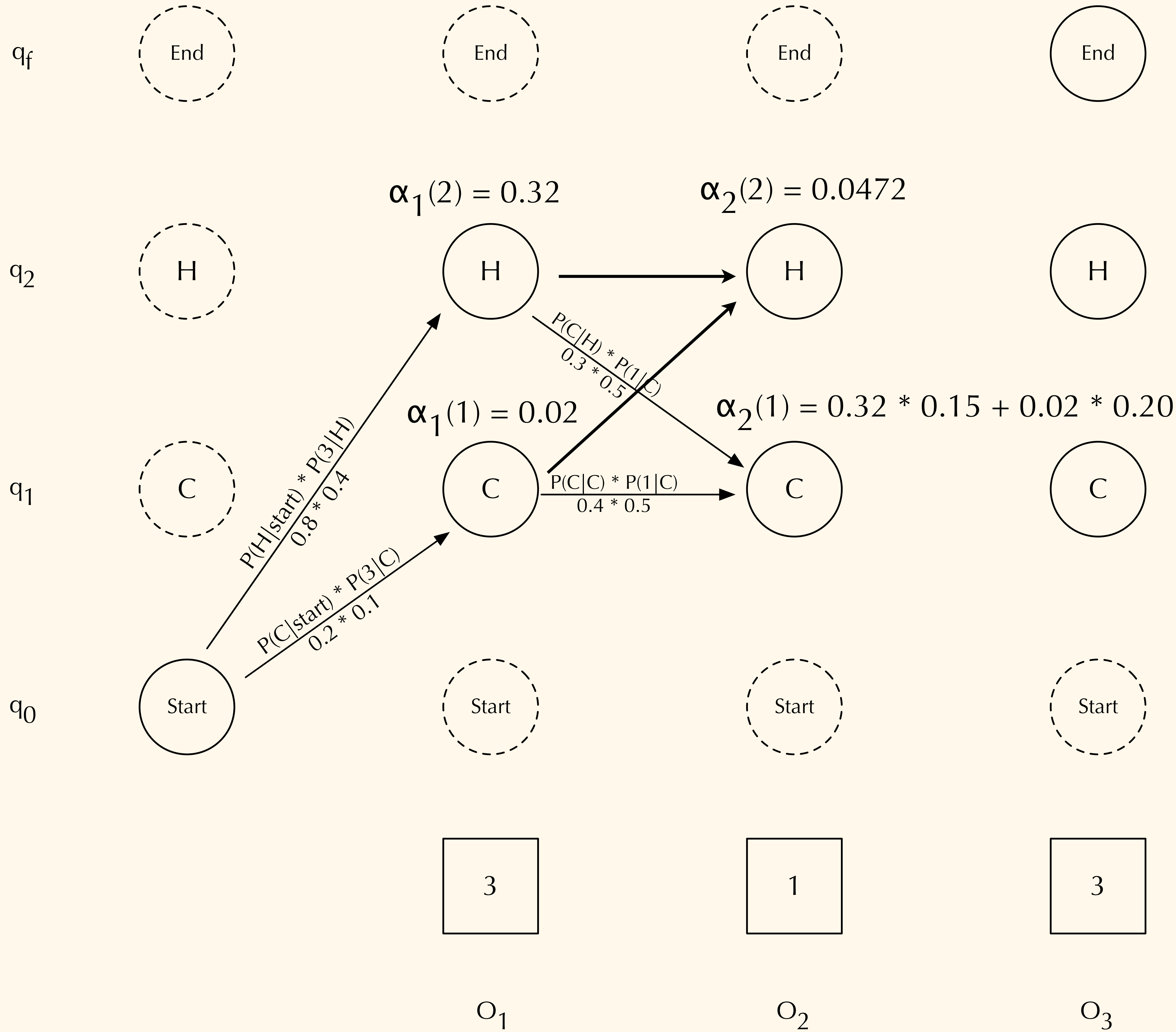
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$\alpha_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(H|C) * P(1|H)
0.4 * 0.2
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



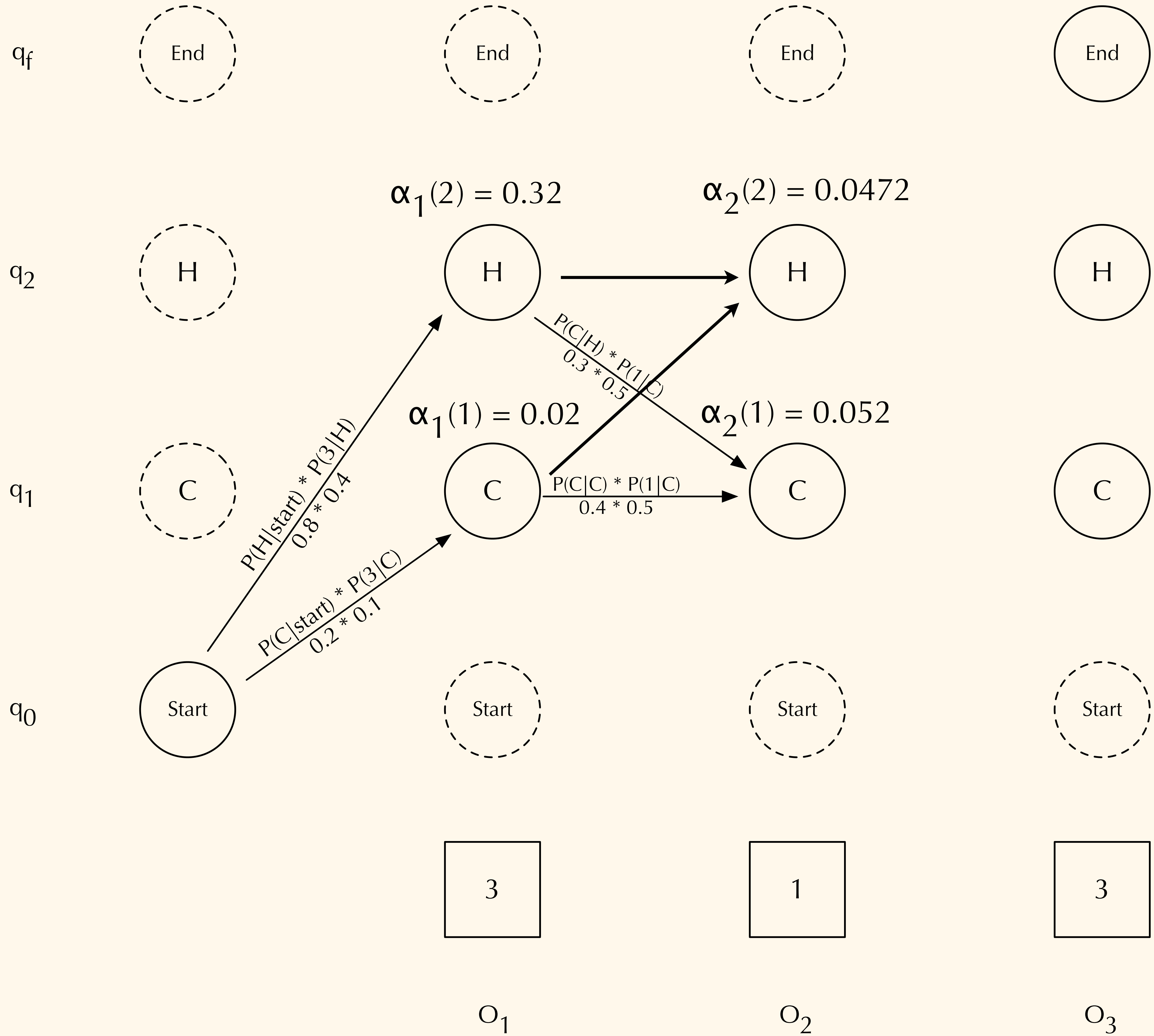
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
$\alpha_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



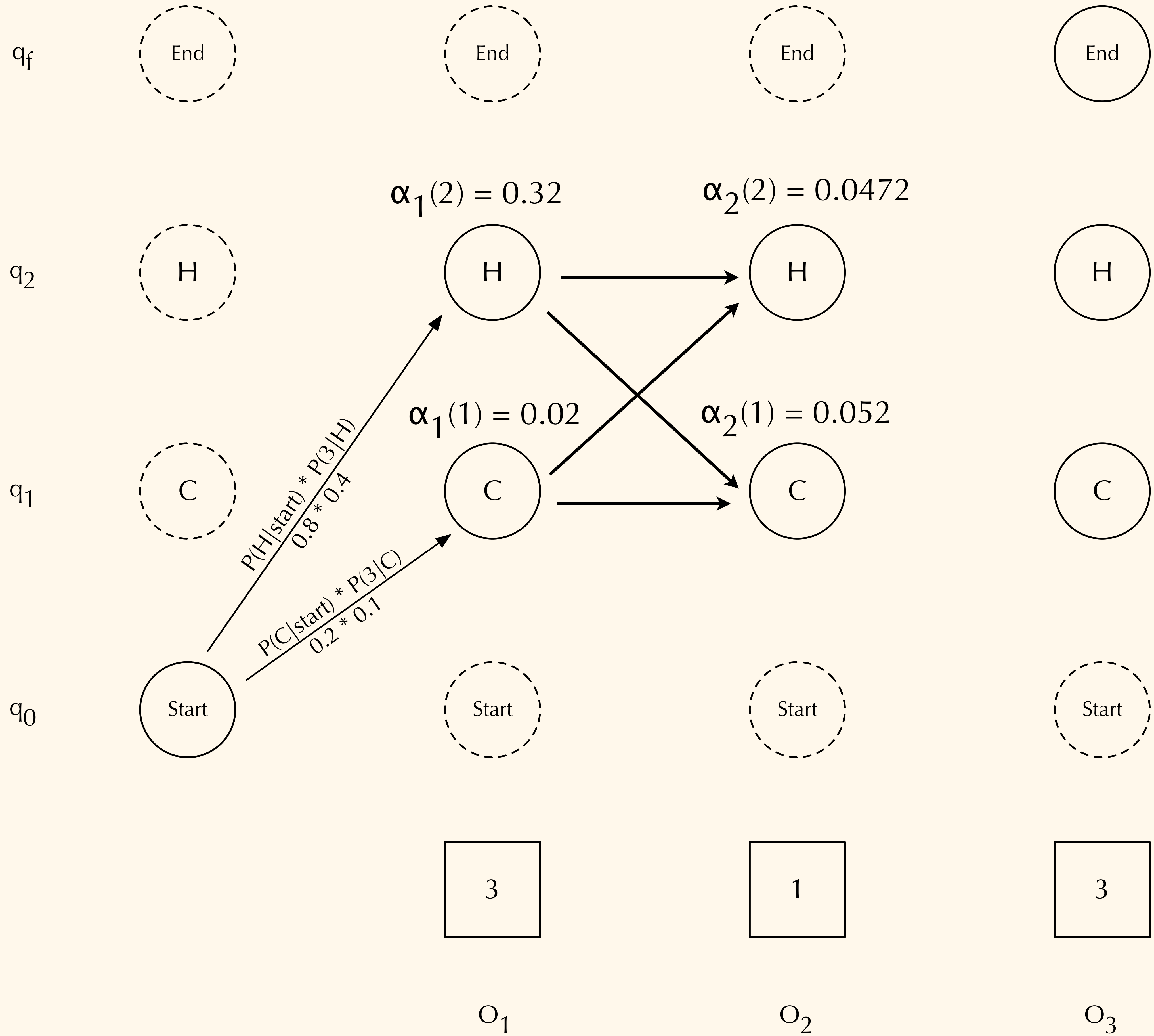
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
P(C|H) * P(1|C)
0.3 * 0.5
$\alpha_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|C) * P(1|C)
0.4 * 0.5
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



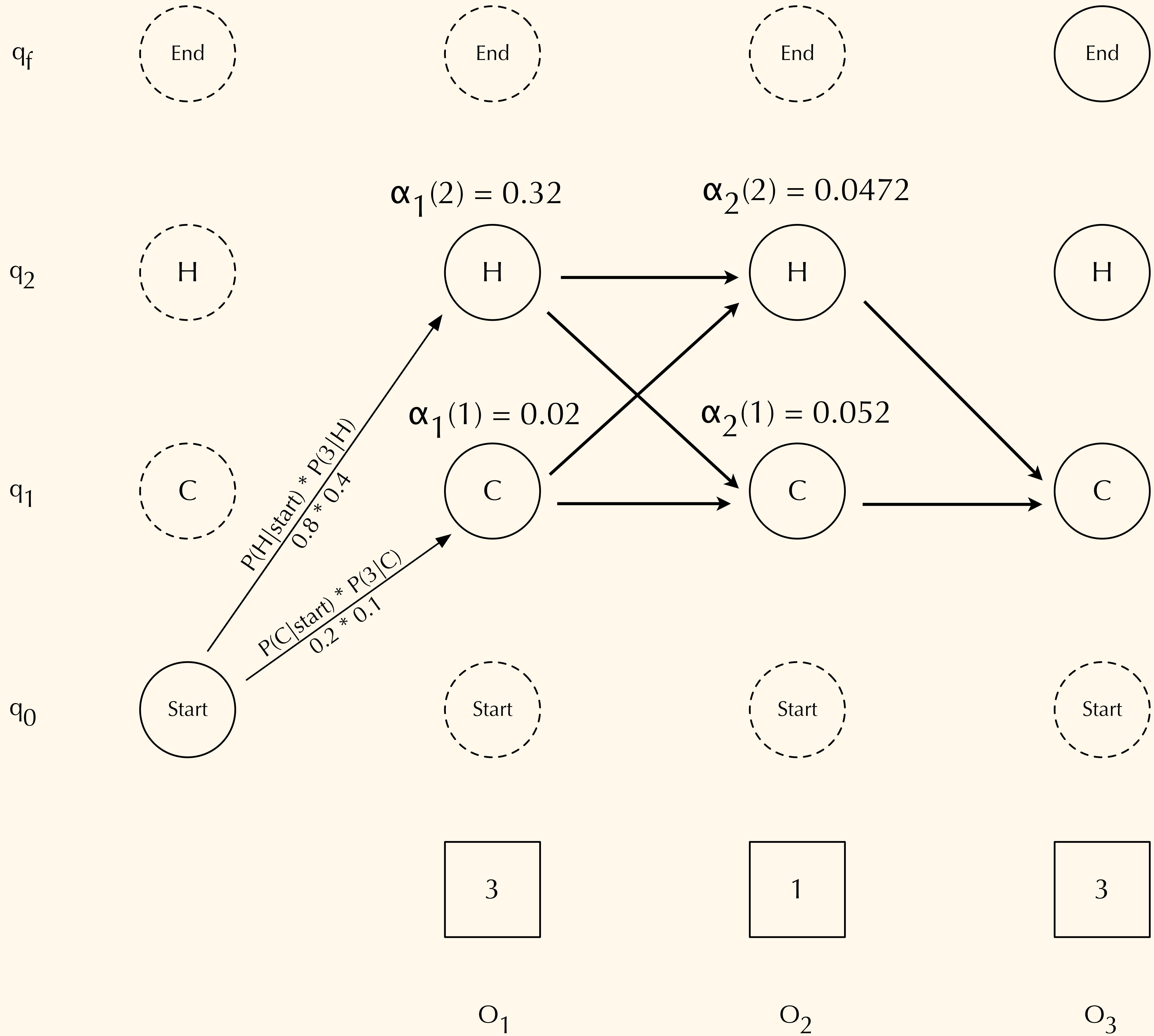
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
P(C|H) * P(1|C)
0.3 * 0.5
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.32 * 0.15 + 0.02 * 0.20$
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|C) * P(1|C)
0.4 * 0.5
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



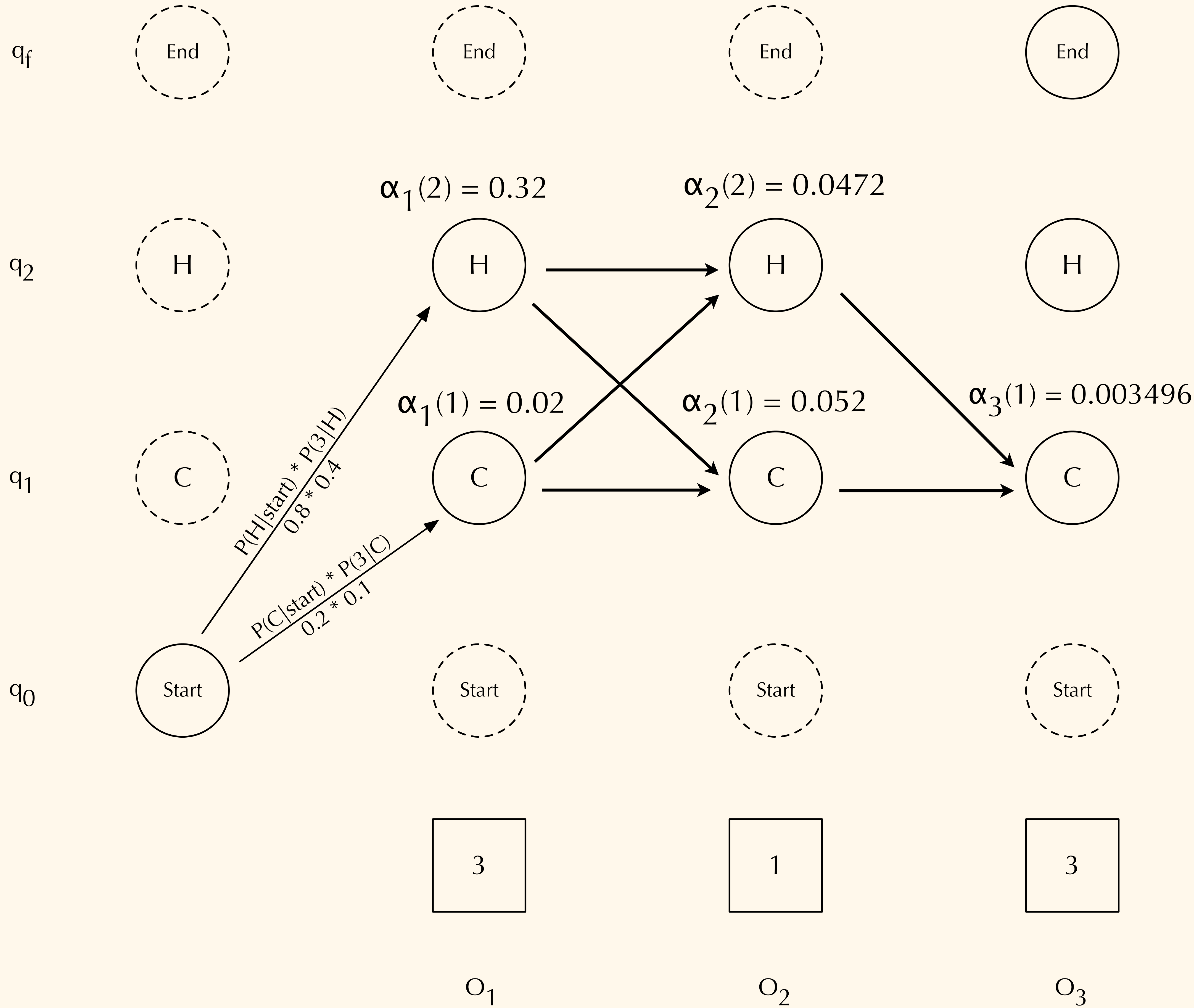
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
P(C|H) * P(1|C)
0.3 * 0.5
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|C) * P(1|C)
0.4 * 0.5
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



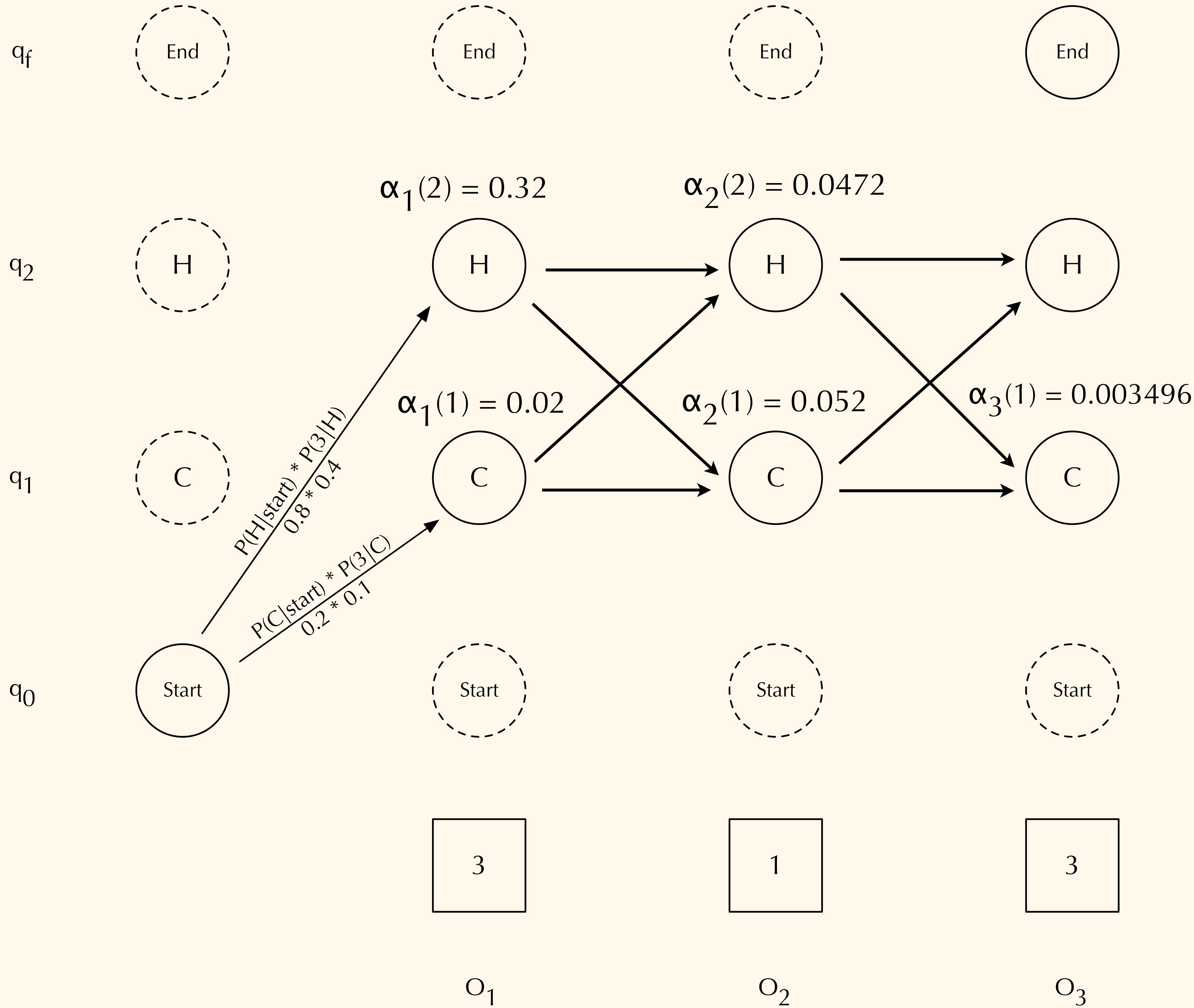
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



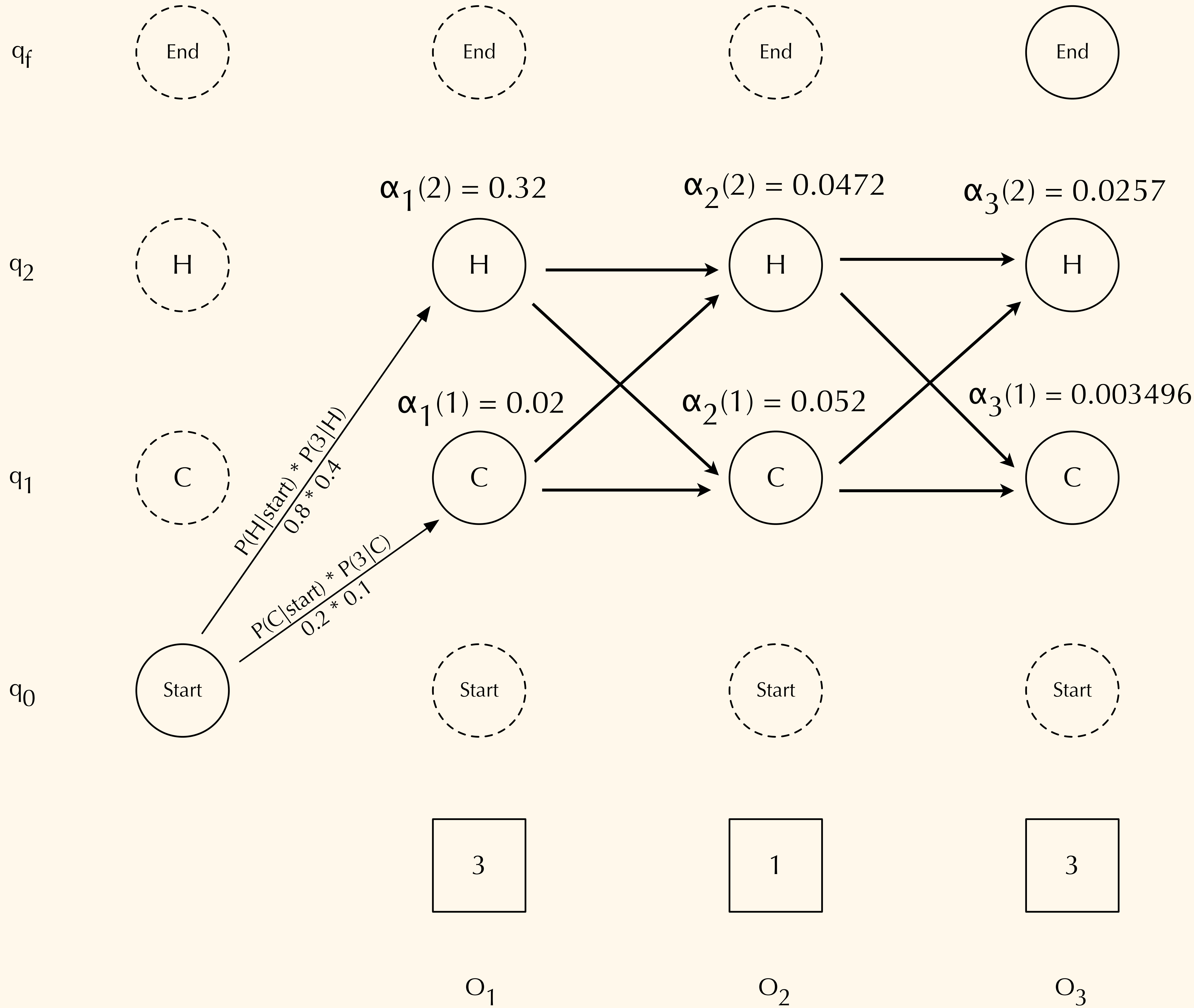
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



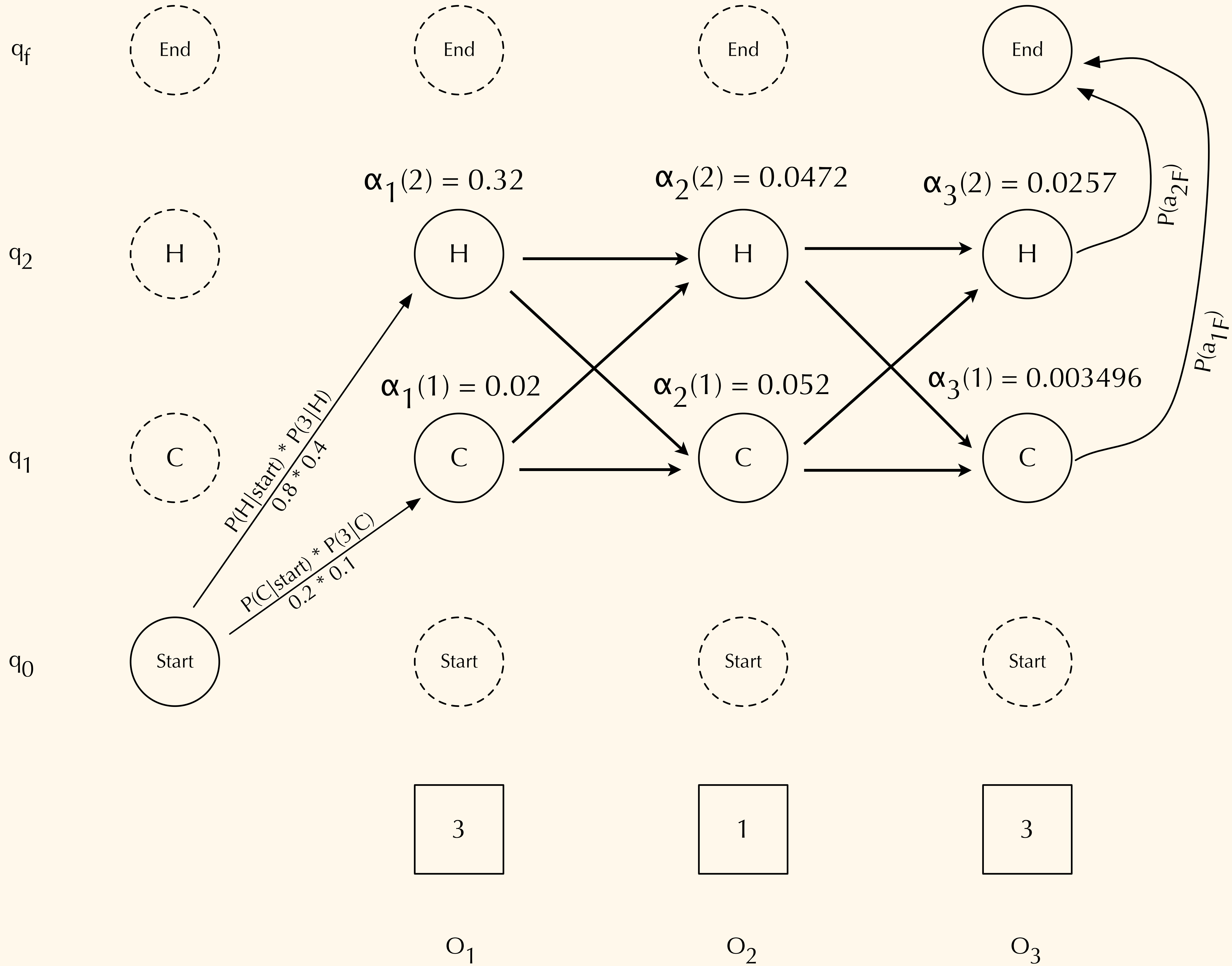
q_f
q_2
q_1
q_0
End
End
End
End
α_1(2) = 0.32
α_2(2) = 0.0472
H
H
H
H
α_1(1) = 0.02
α_2(1) = 0.052
α_3(1) = 0.003496
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



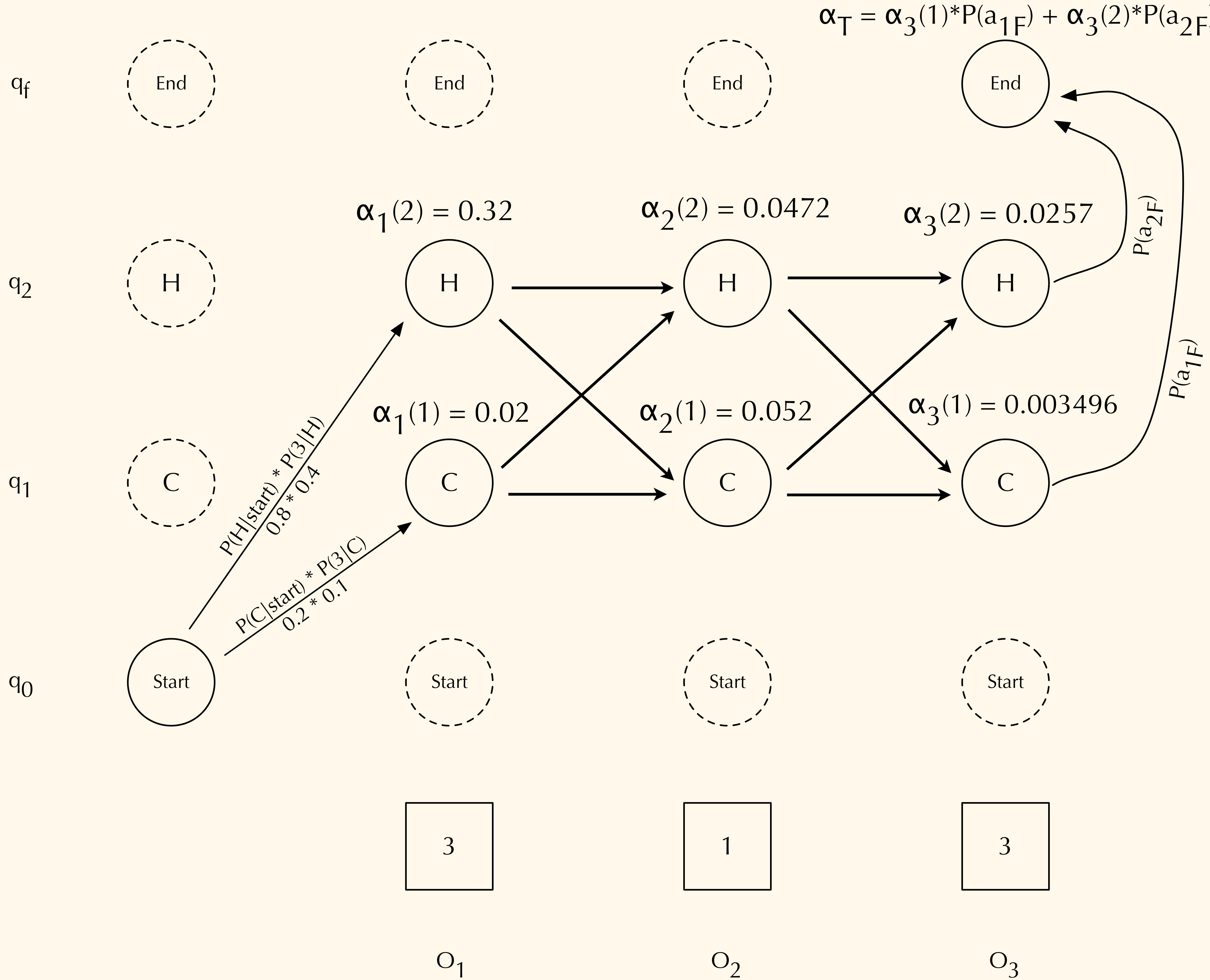
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
H
H
H
H
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
$\alpha_3(1) = 0.003496$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



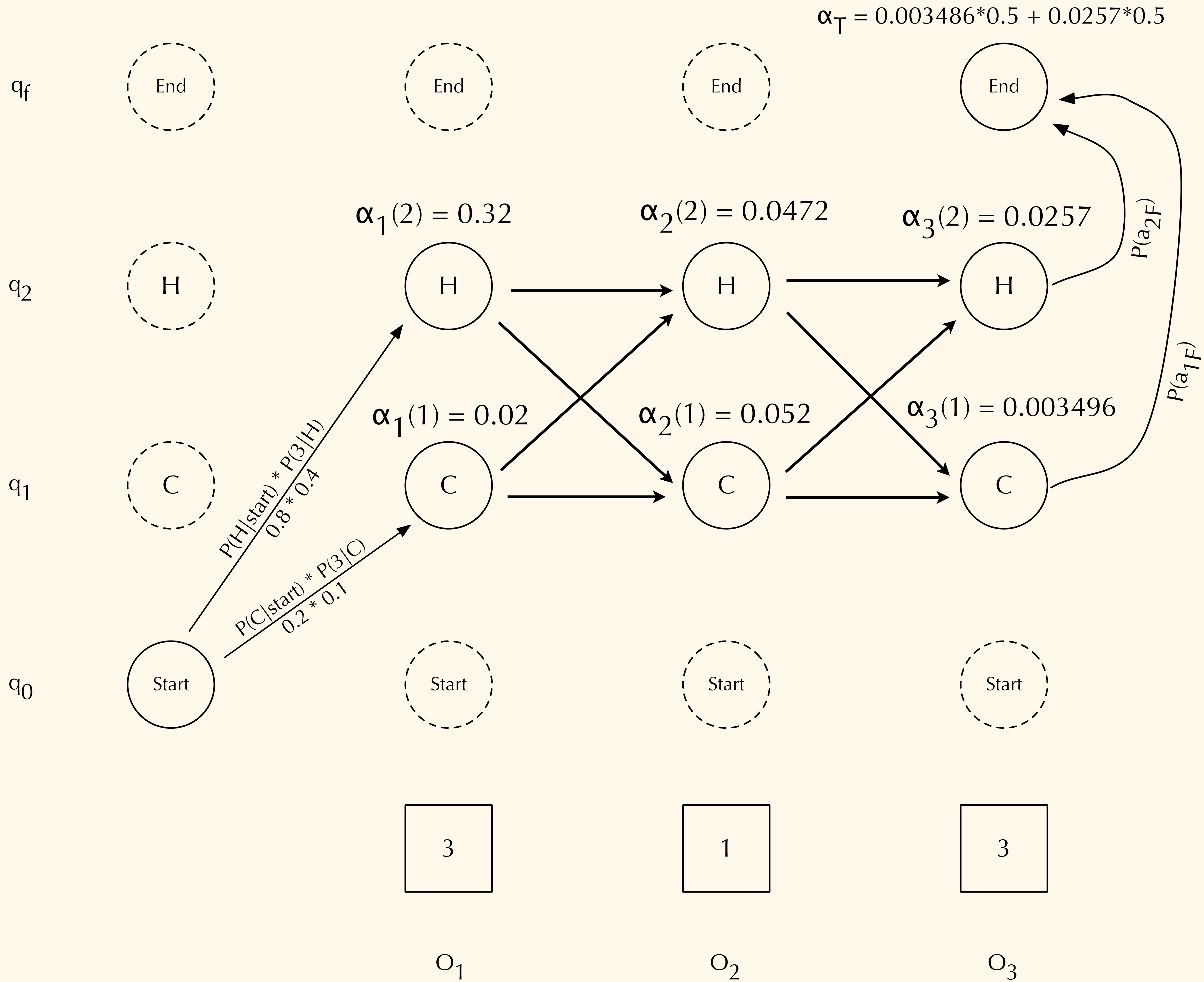
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
$\alpha_3(2) = 0.0257$
H
H
H
H
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
$\alpha_3(1) = 0.003496$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3

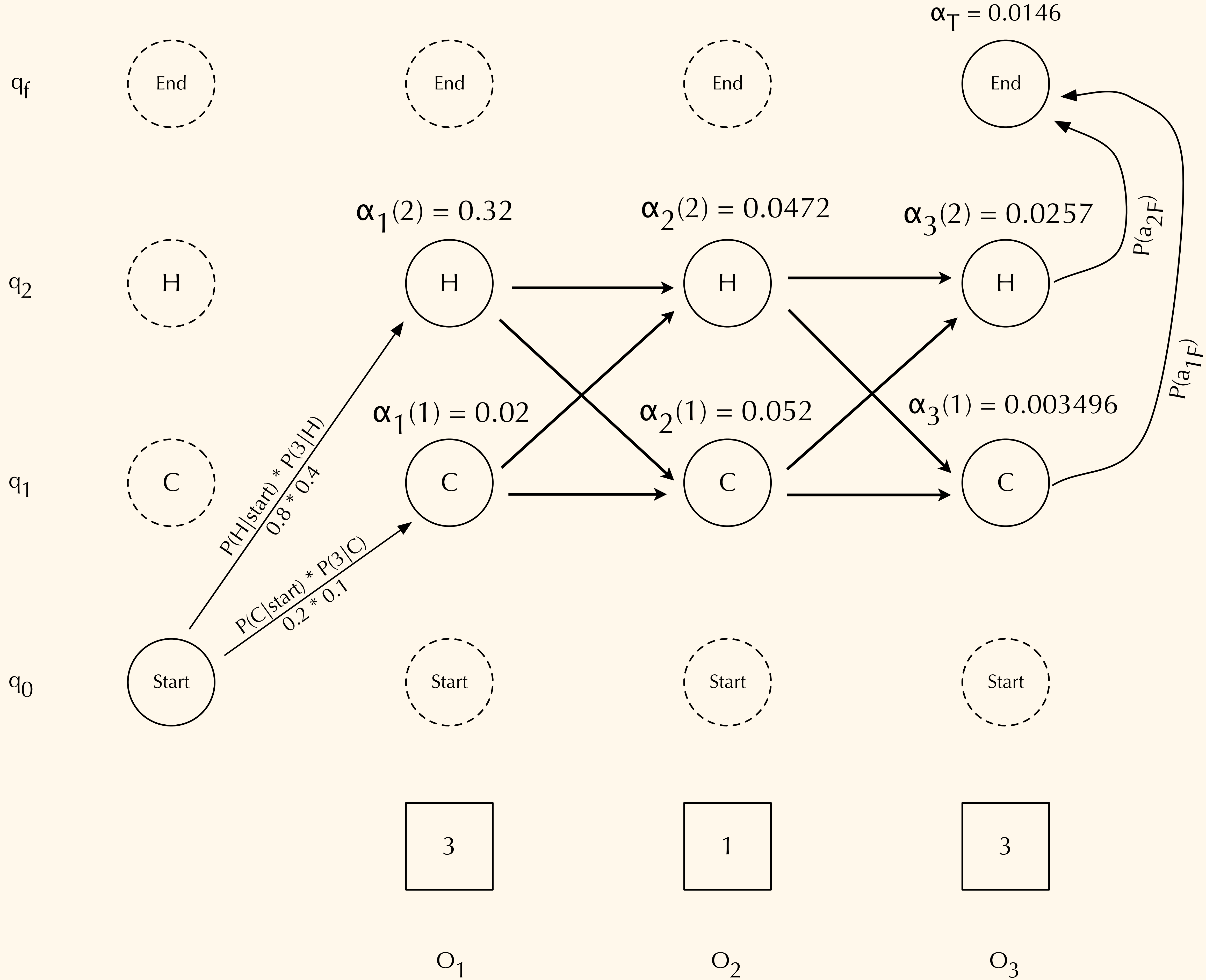


q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
$\alpha_3(2) = 0.0257$
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
$\alpha_3(1) = 0.003496$
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
$P(a_{2F})$
$P(a_{1F})$
3
1
3
O_1
O_2
O_3



$\alpha_T = \alpha_3(1)*P(a_{1F}) + \alpha_3(2)*P(a_{2F})$
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
$\alpha_3(2) = 0.0257$
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
$\alpha_3(1) = 0.003496$
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
$P(a_{2F})$
$P(a_{1F})$
3
1
3
O_1
O_2
O_3


α_T = 0.003486*0.5 + 0.0257*0.5
q_f
q_2
q_1
q_0
End
End
End
End
$\alpha_1(2)$ = 0.32
$\alpha_2(2)$ = 0.0472
$\alpha_3(2)$ = 0.0257
H
H
H
H
$P(a_{2F})$
$P(a_{1F})$
$\alpha_1(1)$ = 0.02
$\alpha_2(1)$ = 0.052
$\alpha_3(1)$ = 0.003496
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



$\alpha_T = 0.0146$
q_f
End
End
End
End
$\alpha_1(2) = 0.32$
$\alpha_2(2) = 0.0472$
$\alpha_3(2) = 0.0257$
$P(a_{2F})$
q_2
H
H
H
H
$P(a_{1F})$
$\alpha_1(1) = 0.02$
$\alpha_2(1) = 0.052$
$\alpha_3(1) = 0.003496$
q_1
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
q_0
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3

There are three fundamental kinds of questions that we can ask with an HMM:

1. ***Likelihood:* Given a sequence of states, what is the most likely observed sequence? *or* How likely is a given observation sequence?**

2. *Decoding:* Given an observation sequence and a fully-specified HMM, what is the most likely sequence of states to have produced that observation?

3. *Learning:* Given an observation sequence and a set of states, what are the likely transition and emission probabilities (A and B)?

There are three fundamental kinds of questions that we can ask with an HMM:

1. *Likelihood:* Given a sequence of states, what is the most likely observed sequence? *or* How likely is a given observation sequence?

2. ***Decoding:*** **Given an observation sequence and a fully-specified HMM, what is the most likely sequence of states to have produced that observation?**

3. *Learning:* Given an observation sequence and a set of states, what are the likely transition and emission probabilities (A and B)?

Formally: $\operatorname*{argmax}_{Q} P(Q|O)$

Formally: $\underset{Q}{\operatorname{argmax}} P(Q|O)$

"Given an observation *O*, what was the most probable sequence of states Q?"

Formally: $\underset{Q}{\operatorname{argmax}} P(Q|O)$

"Given an observation O, what was the most probable sequence of states Q?"

Decoding and likelihood estimation have certain similarities...

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"Given an observation O, what was the most probable sequence of states Q?"

Decoding and likelihood estimation have certain similarities...

One solution: run the forward algorithm over each possible state sequence...

Formally: $\underset{Q}{\operatorname{argmax}} P(Q|O)$

“Given an observation O, what was the most probable sequence of states Q?”

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... which has the same issue as the naïve solution to the likelihood problem!

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"Given an observation *O*, what was the most probable sequence of states Q?"

Decoding and likelihood estimation have certain similarities...

One solution: run the forward algorithm over each possible state sequence...

... which has the same issue as the naïve solution to the likelihood problem!

$O(N^T)$ possible solutions...

Decoding and likelihood estimation have the same problems...

Decoding and likelihood estimation have the same problems...

... and they share a solution.

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… and they share a solution.

Modifying the forward algorithm slightly gives us the *Viterbi algorithm* for decoding.

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... and they share a solution.

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The main difference: instead of *summing* possible paths to each state, we take the *max*...

Decoding and likelihood estimation have the same problems...

... and they share a solution.

Modifying the forward algorithm slightly gives us the *Viterbi algorithm* for decoding.

The main difference: instead of *summing* possible paths to each state, we take the *max*...

... and *keep track* of which one it was!

Forward algorithm trellis locations:

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

Forward algorithm trellis locations:

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

Viterbi algorithm trellis locations:

$$v_t(j) = \max_{i=1}^{N} v_{t-1} a_{ij} b_j(o_t)$$

Forward algorithm trellis locations:

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

Viterbi algorithm trellis locations:

$$v_t(j) = \max_{i=1}^{N} v_{t-1} a_{ij} b_j(o_t)$$

We also save a backtrace through the most-likely states:

Forward algorithm trellis locations:

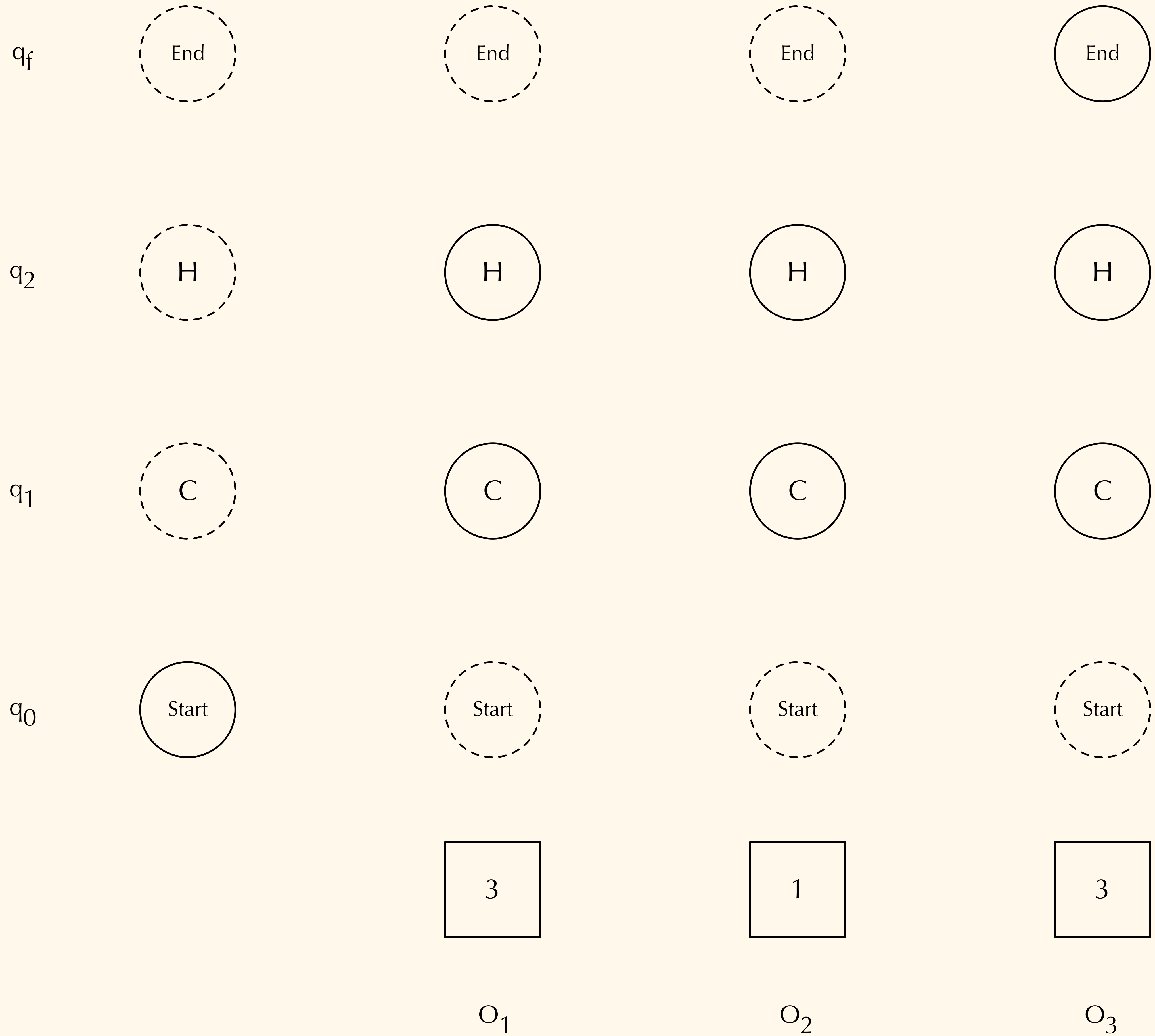
$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

Viterbi algorithm trellis locations:

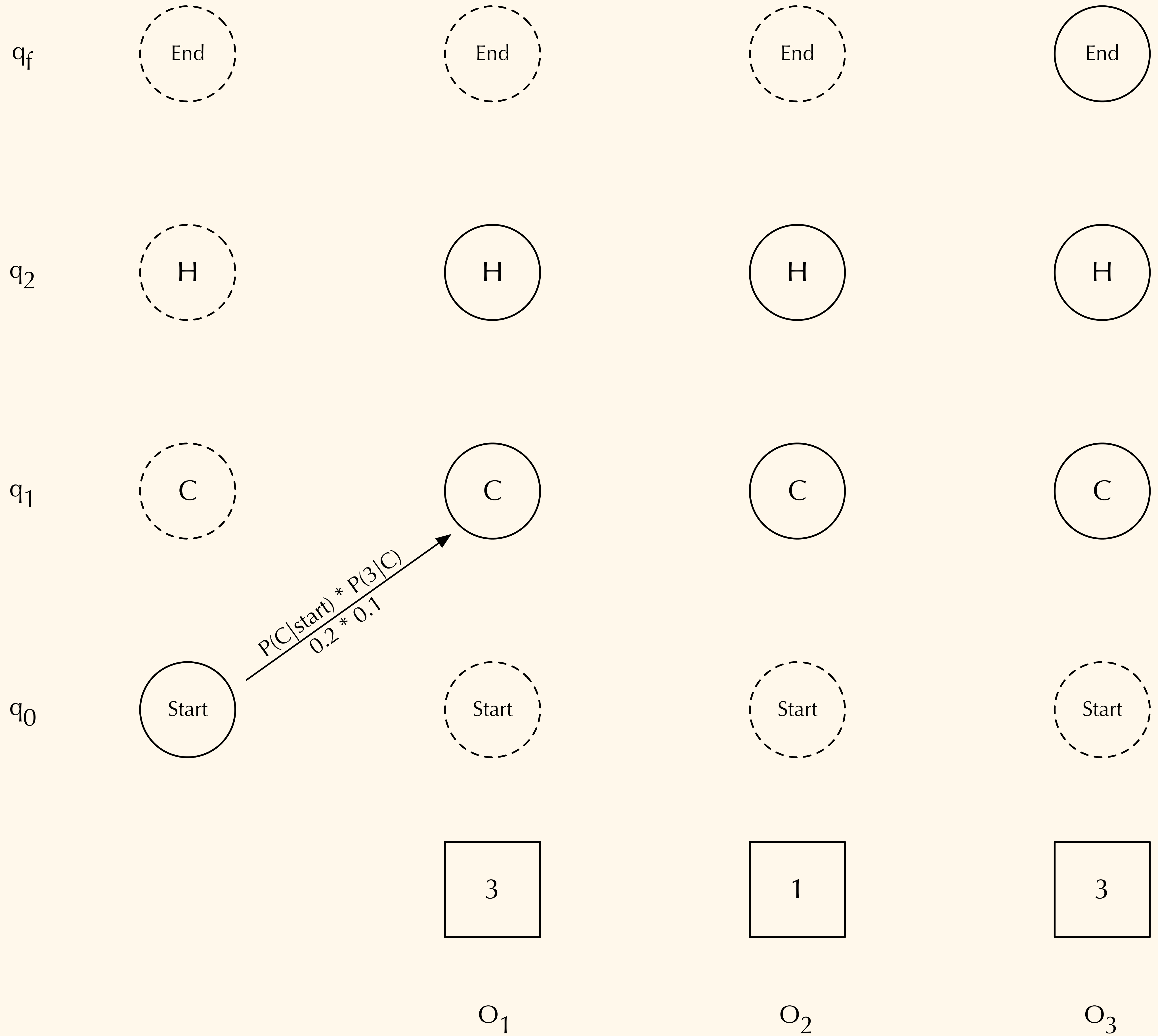
$$v_t(j) = \max_{i=1}^{N} v_{t-1} a_{ij} b_j(o_t)$$

We also save a backtrace through the most-likely states:

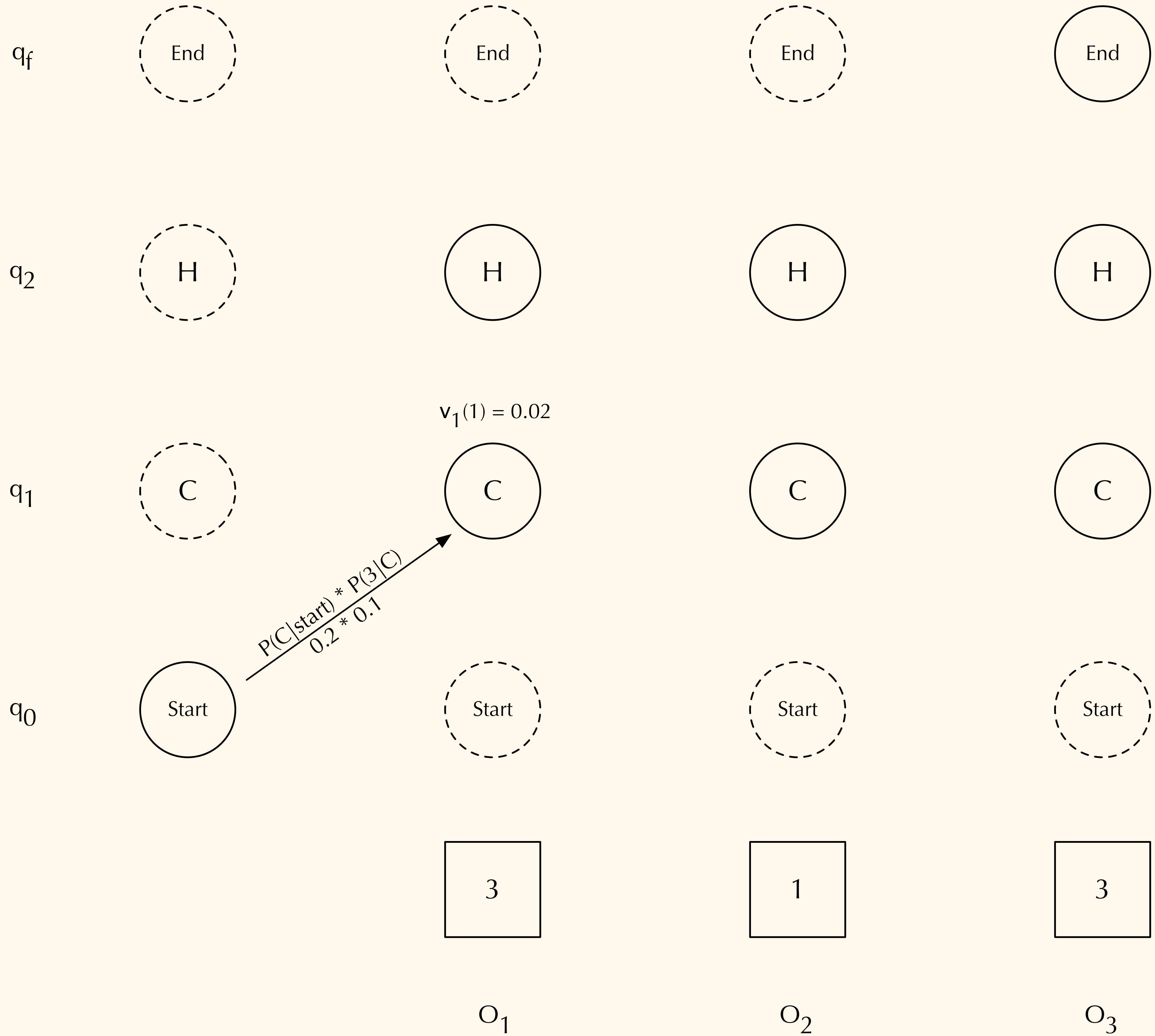
$$bt_t(j) = \operatorname*{argmax}_{i=1}^{N} v_{t-1} a_{ij} b_j(o_t)$$



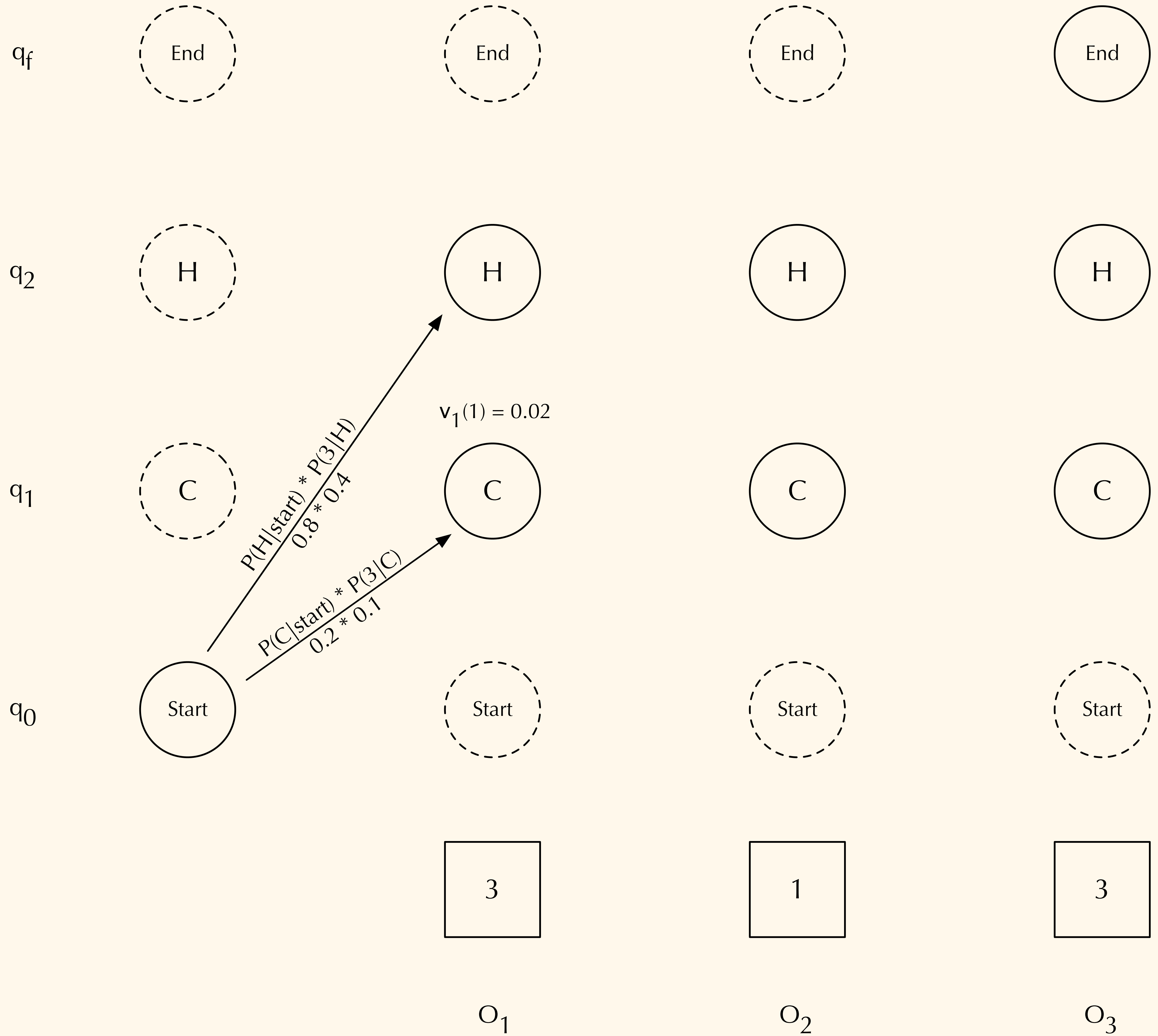
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



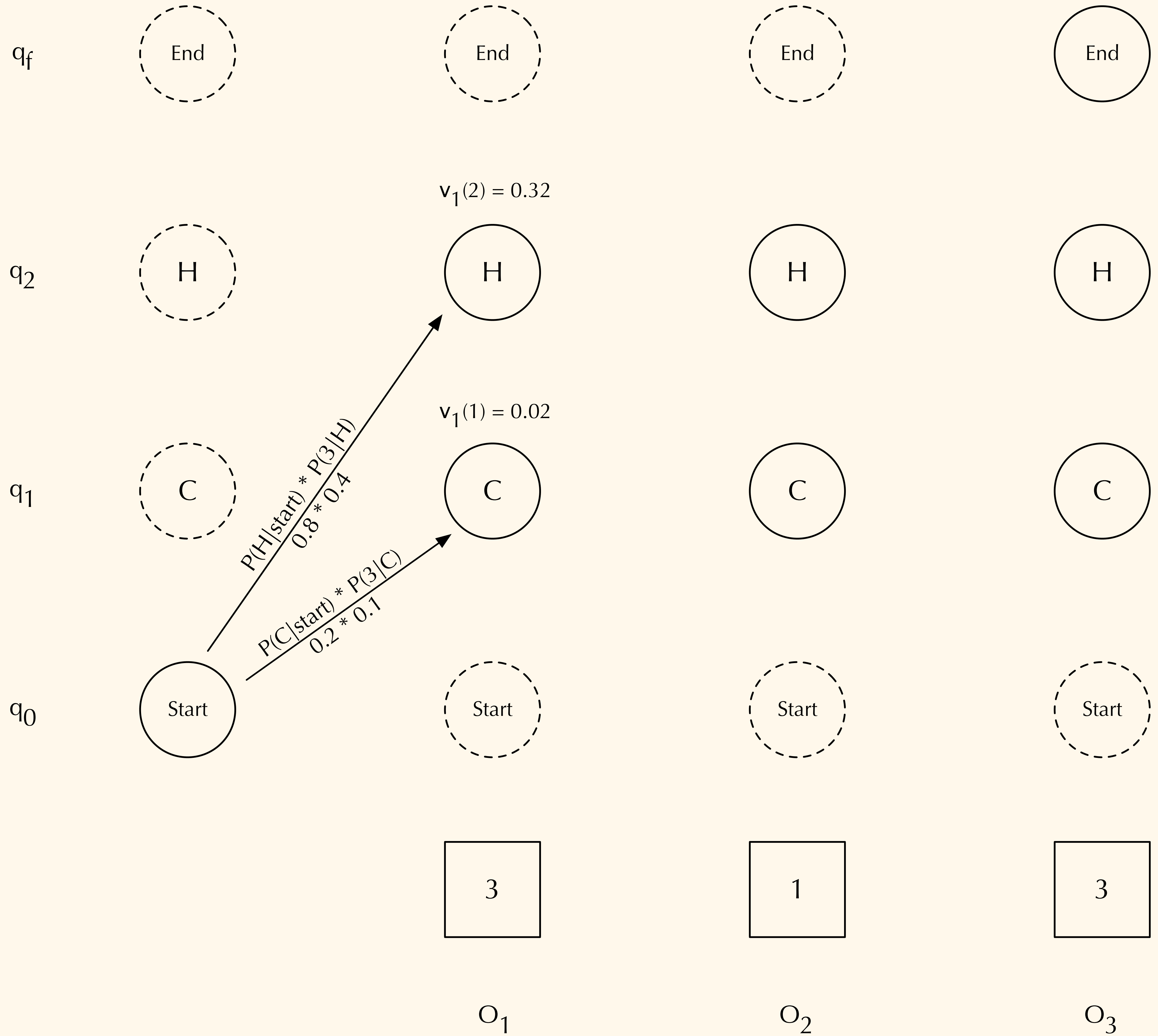
qf
q2
q1
q0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
P(C|start) * P(3|C)
0.2 * 0.1
3
1
3
O1
O2
O3



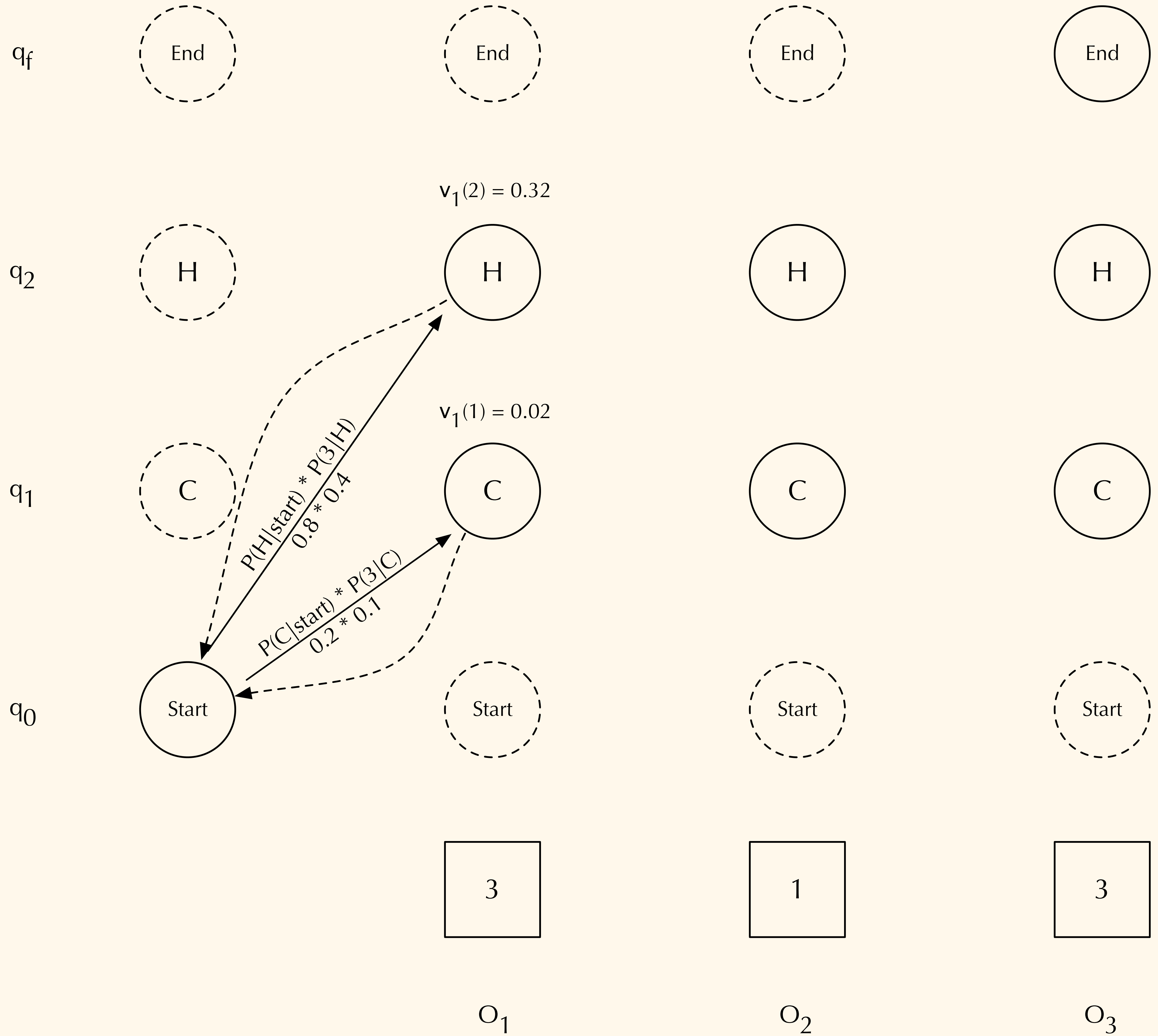
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
v_1(1) = 0.02
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



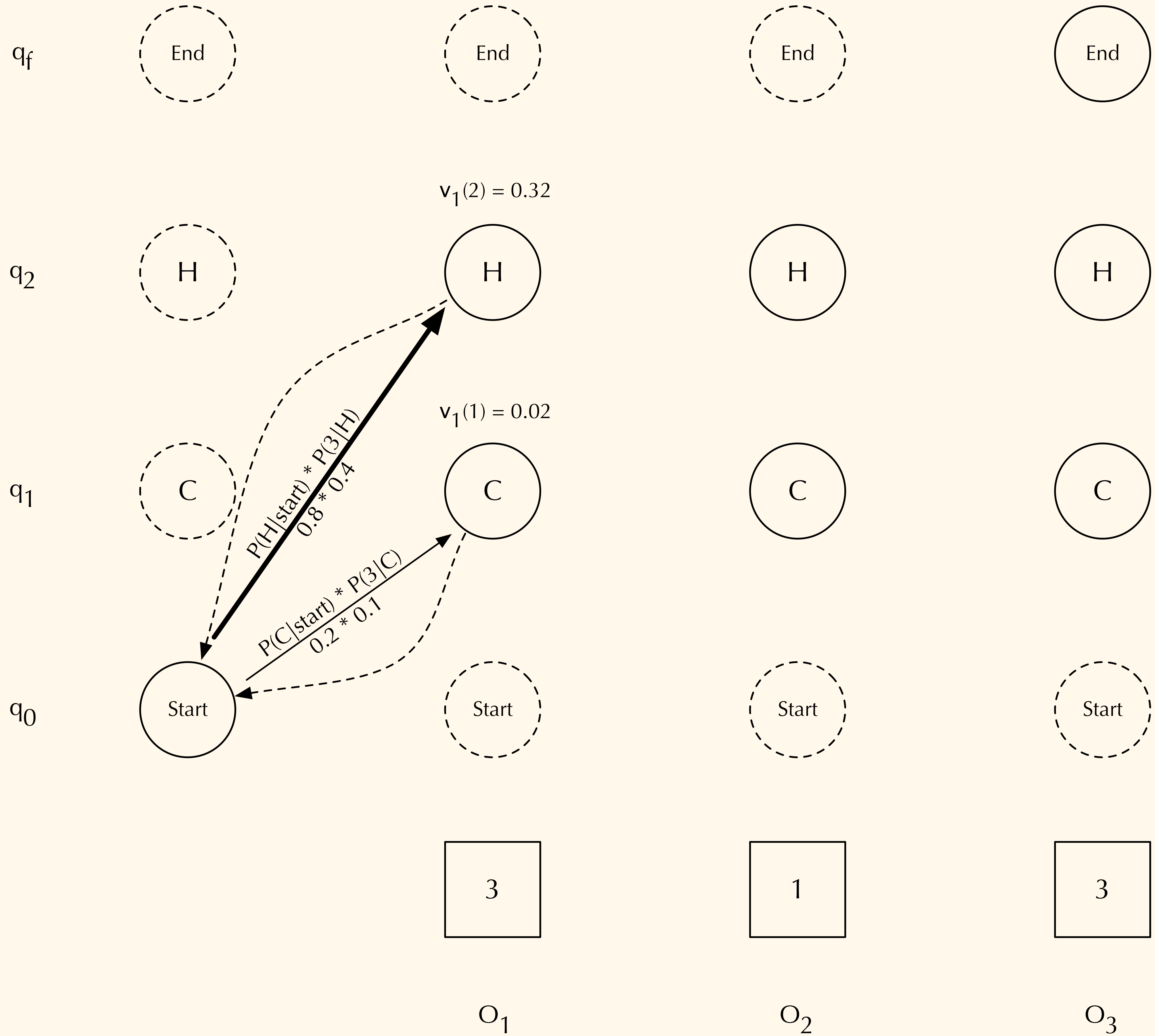
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
v_1(1) = 0.02
3
1
3
O_1
O_2
O_3

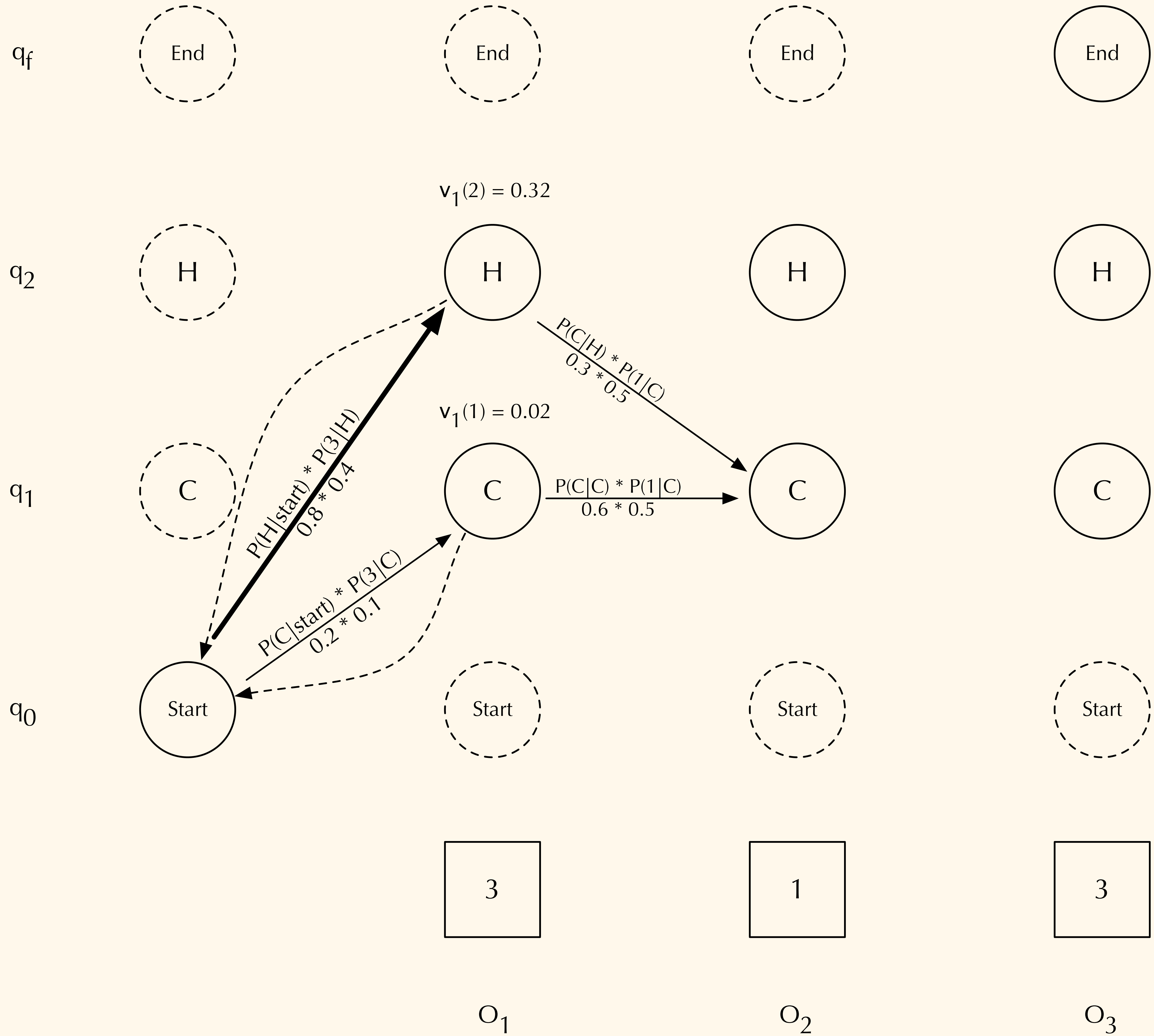


q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
H
H
H
H
$v_1(1) = 0.02$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3

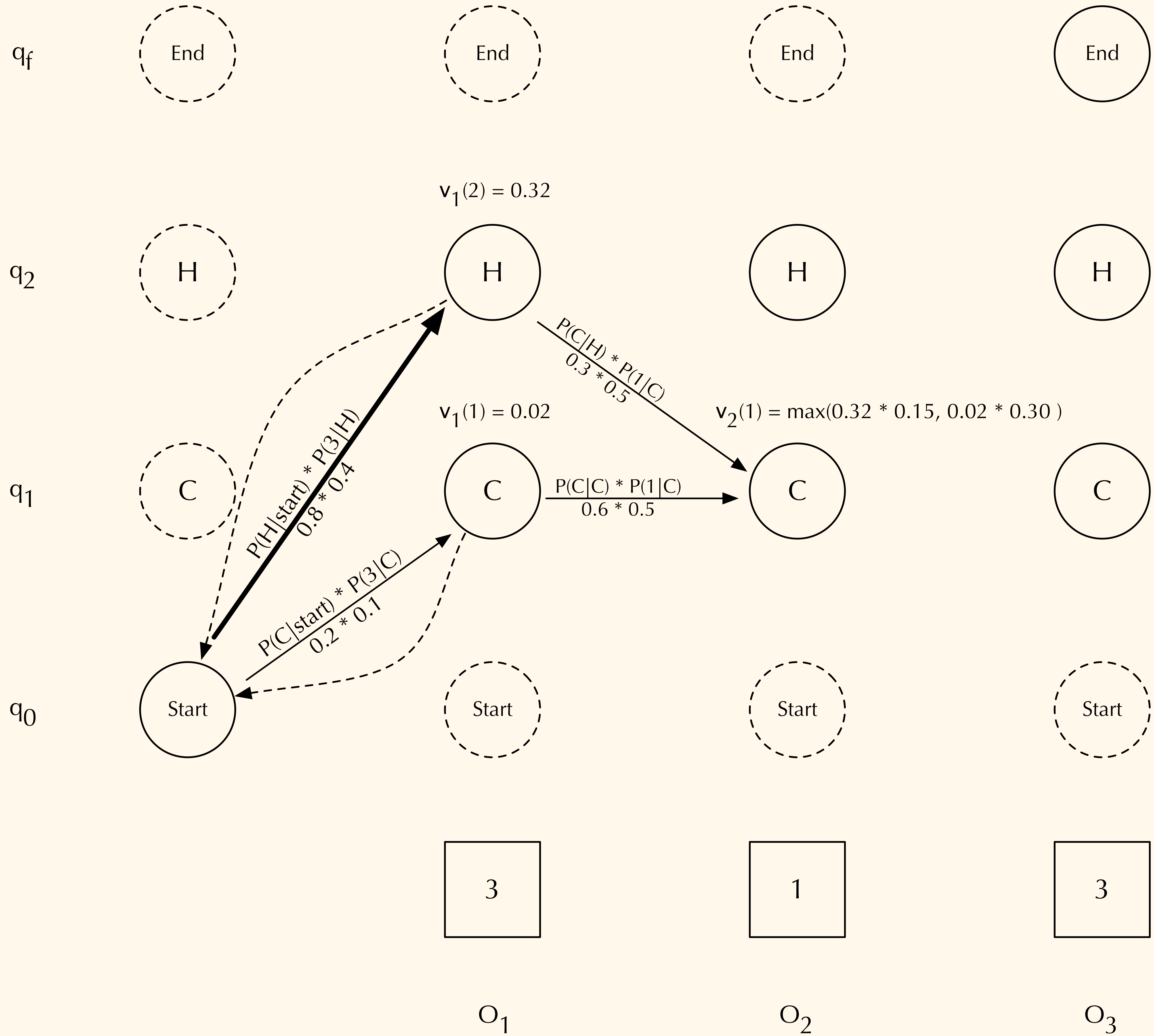


q_f
q_2
q_1
q_0
End
End
End
End
v_1(2) = 0.32
H
H
H
H
v_1(1) = 0.02
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3

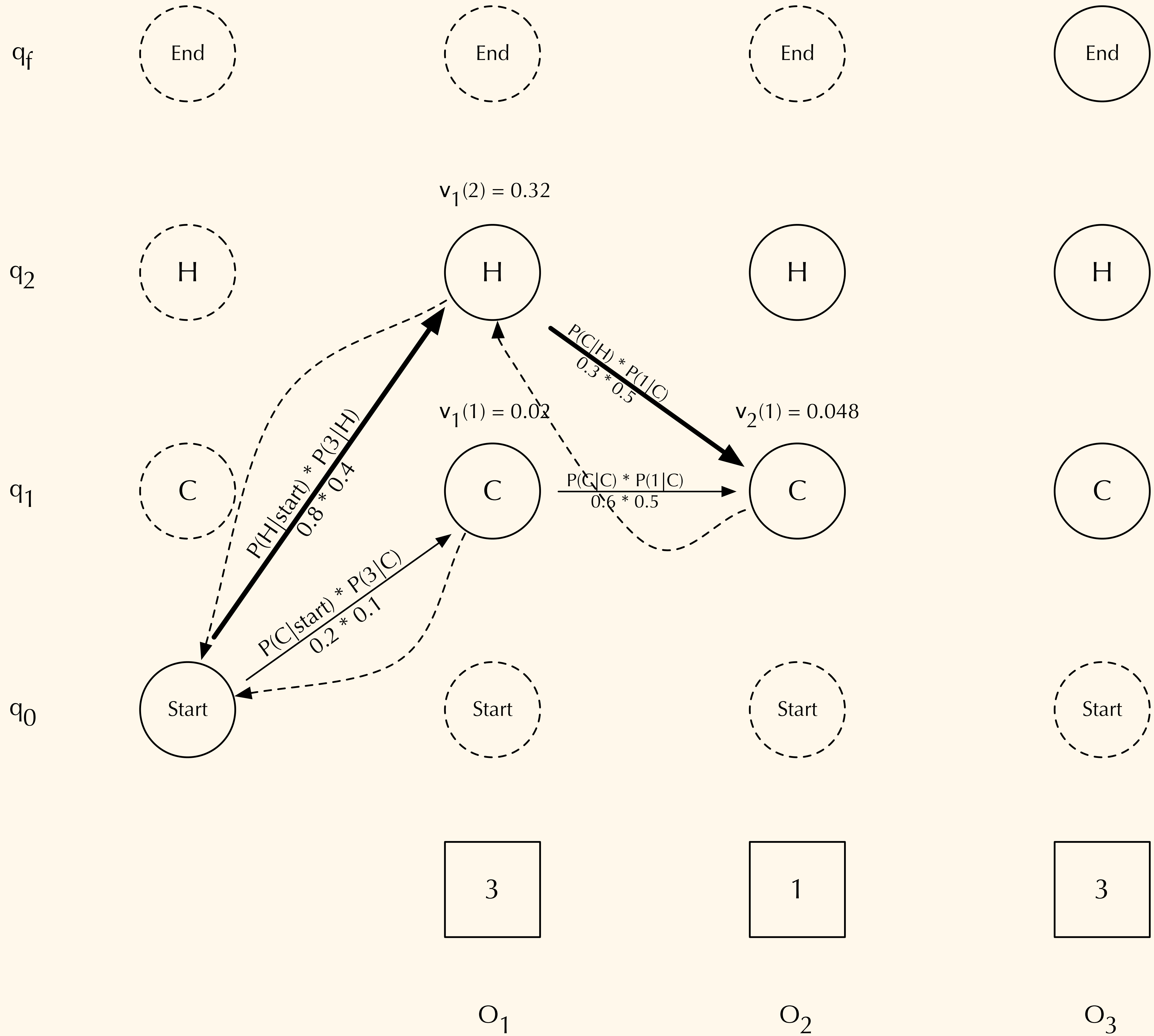

q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
H
H
H
H
$v_1(1) = 0.02$
C
C
C
C
Start
Start
Start
Start
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
3
1
3
O_1
O_2
O_3



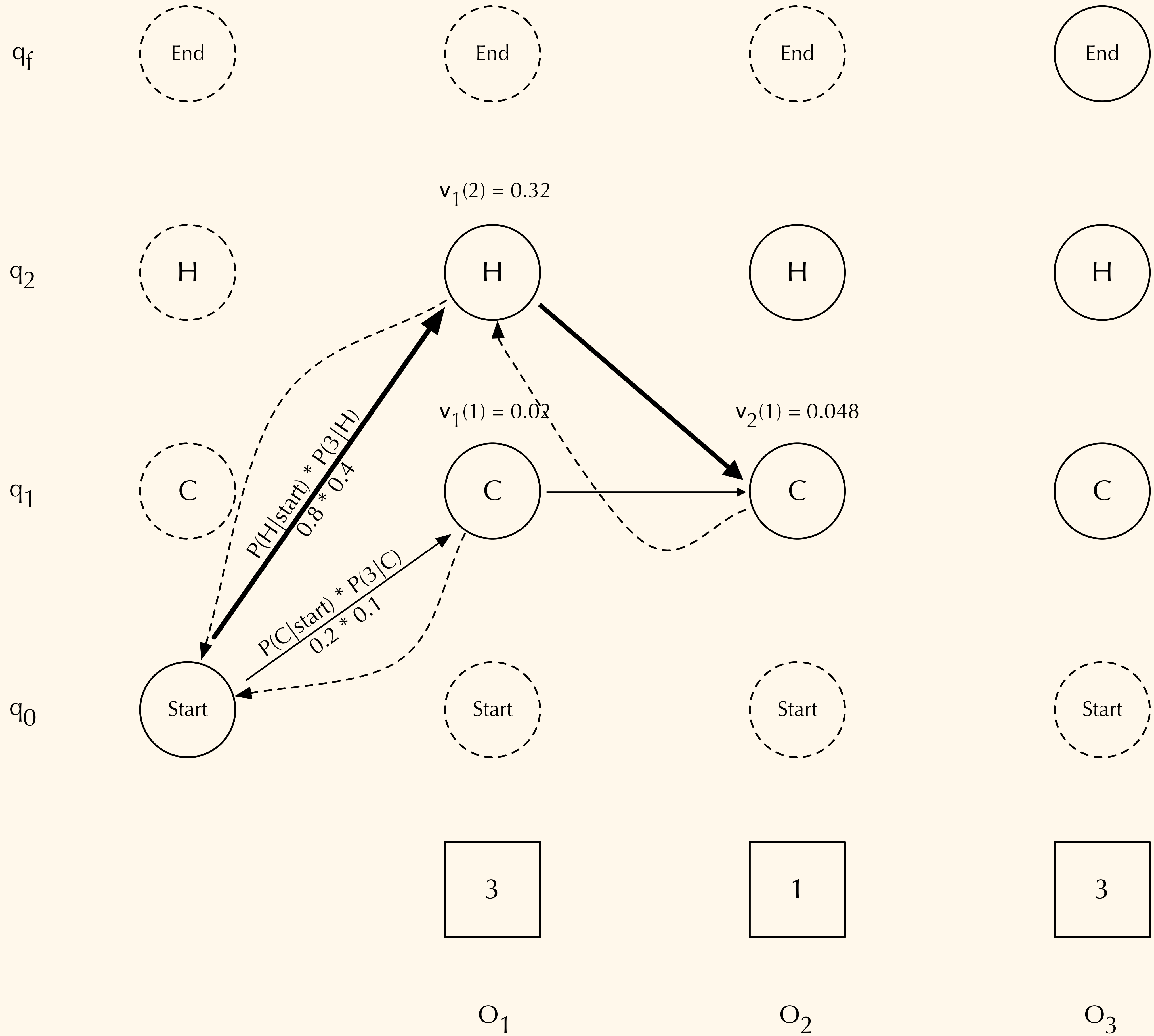
q_f
q_2
q_1
q_0
End
End
End
End
v_1(2) = 0.32
H
H
H
H
P(C|H) * P(1|C)
0.3 * 0.5
v_1(1) = 0.02
P(H|start) * P(3|H)
0.8 * 0.4
C
C
P(C|C) * P(1|C)
0.6 * 0.5
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



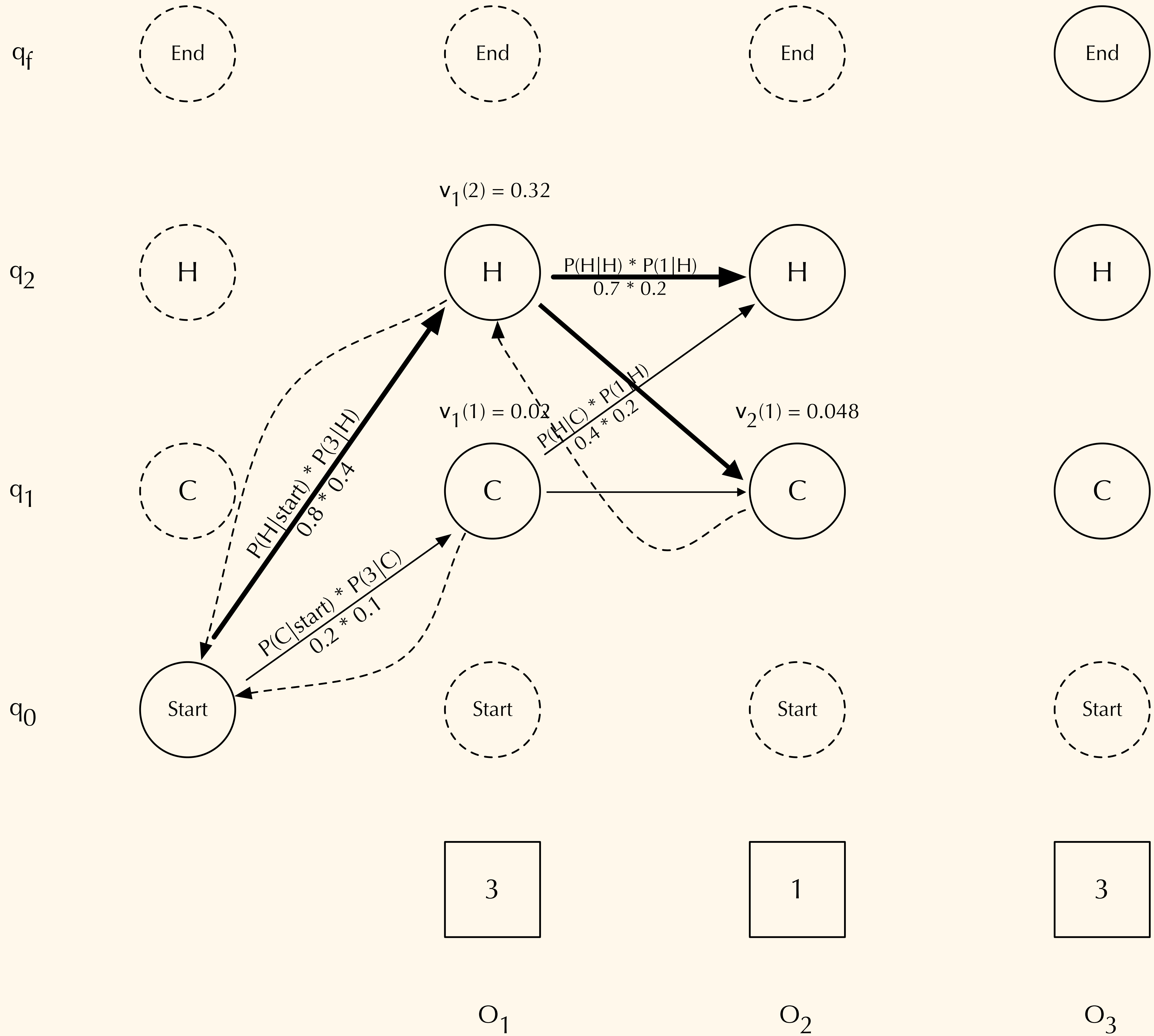
q_f
q_2
q_1
q_0
End
End
End
End
v_1(2) = 0.32
H
H
H
H
P(C|H) * P(1|C)
0.3 * 0.5
v_1(1) = 0.02
v_2(1) = max(0.32 * 0.15, 0.02 * 0.30)
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|C) * P(1|C)
0.6 * 0.5
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



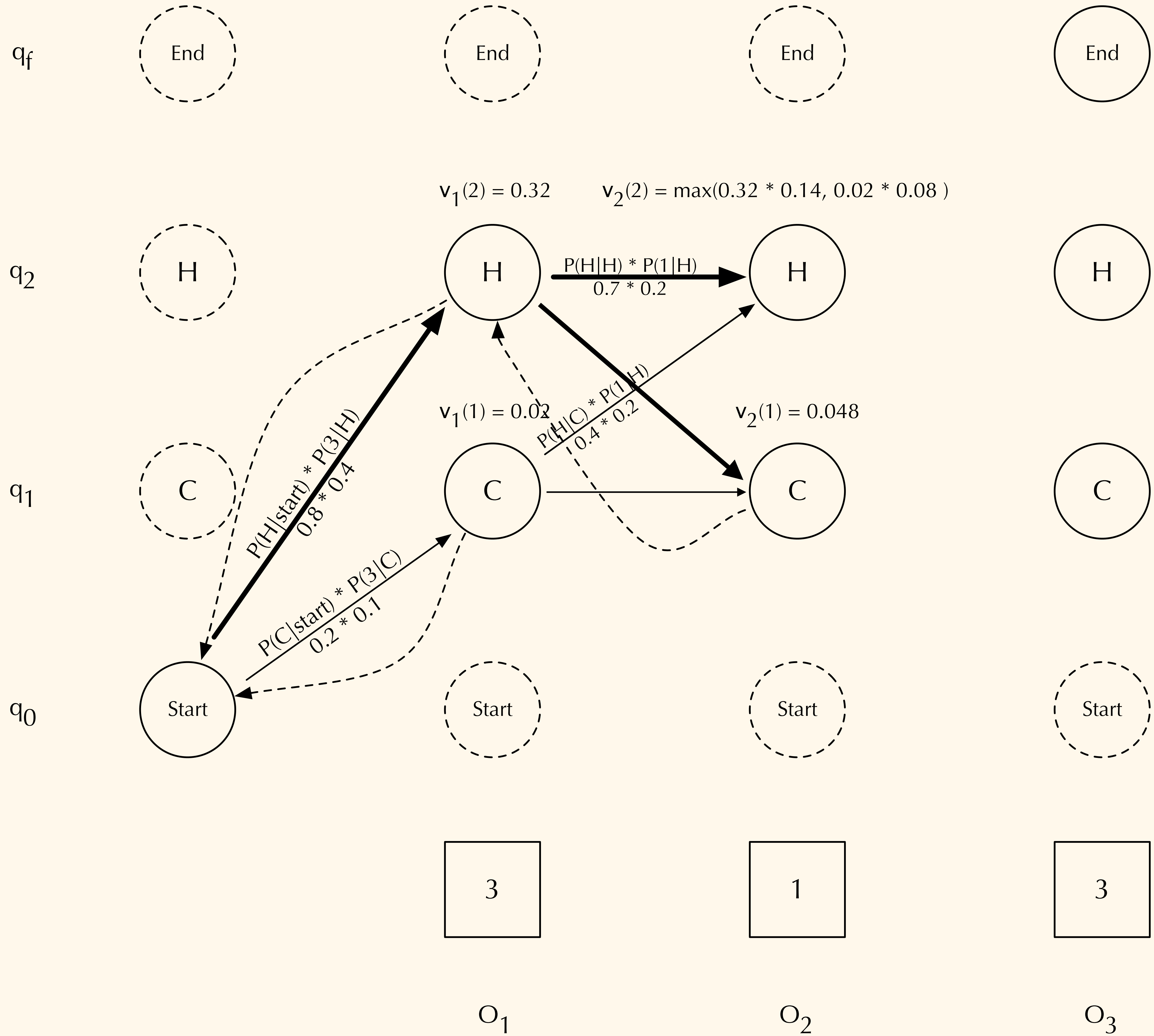
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
H
H
H
H
$v_1(1) = 0.02$
$v_2(1) = 0.048$
C
C
C
C
Start
Start
Start
Start
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
P(C|H) * P(1|C)
0.3 * 0.5
P(C|C) * P(1|C)
0.6 * 0.5
3
1
3
O_1
O_2
O_3



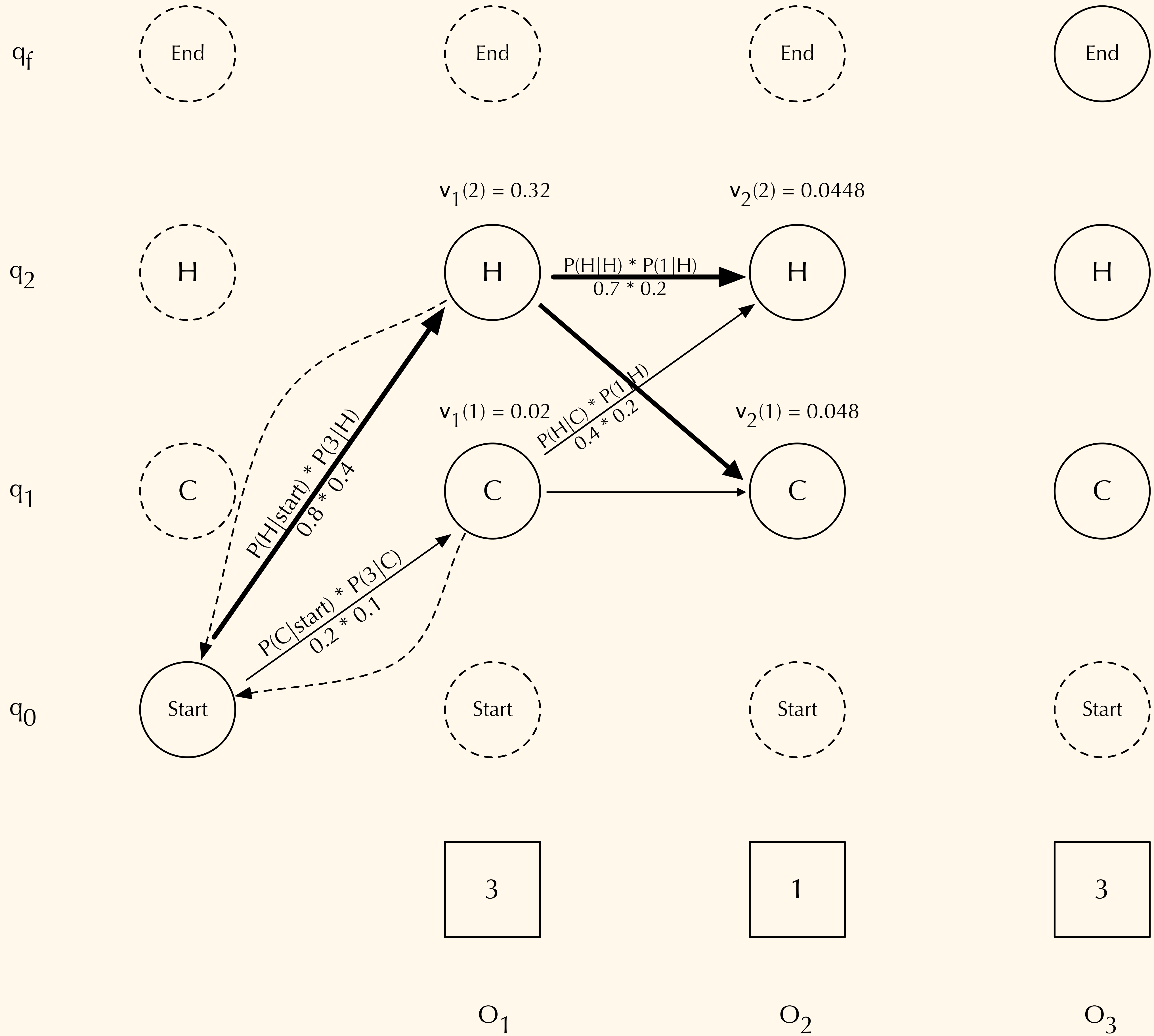
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
H
H
H
H
$v_1(1) = 0.02$
$v_2(1) = 0.048$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



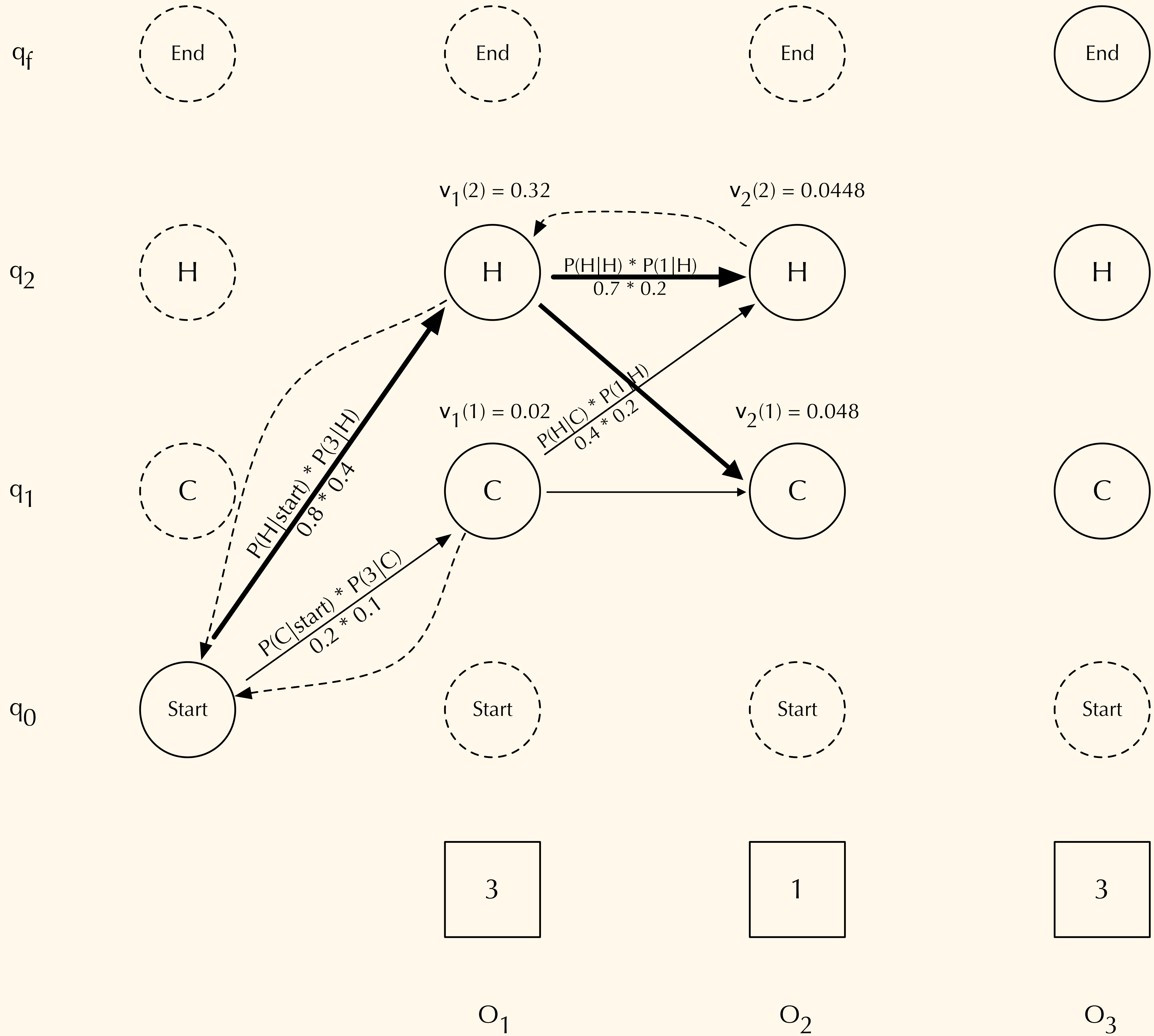
q_f
q_2
q_1
q_0
End
End
End
End
v_1(2) = 0.32
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
v_1(1) = 0.02
v_2(1) = 0.048
P(H|C) * P(1|H)
0.4 * 0.2
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



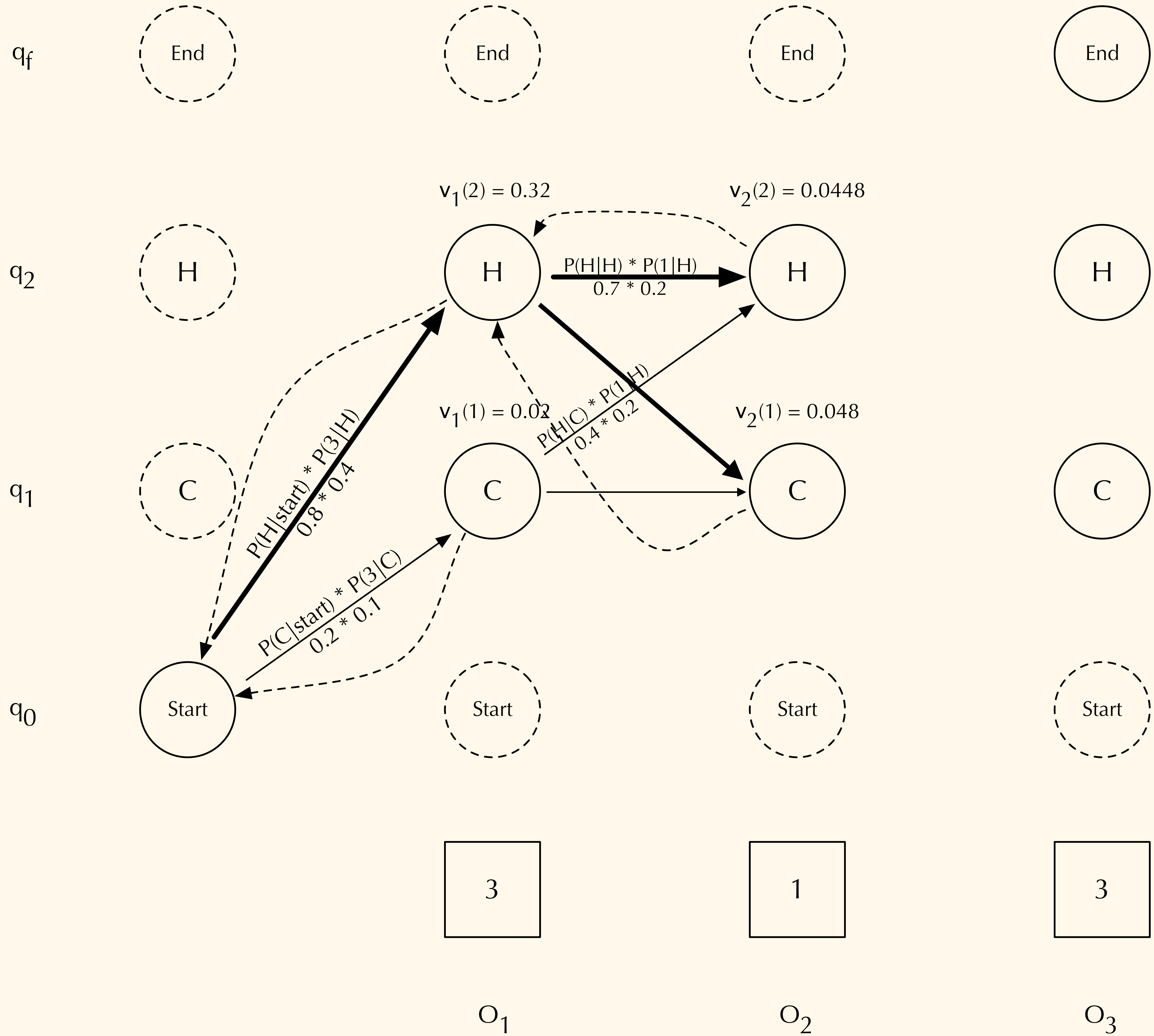
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
v_1(2) = 0.32
v_2(2) = max(0.32 * 0.14, 0.02 * 0.08)
v_1(1) = 0.02
v_2(1) = 0.048
P(H|H) * P(1|H)
0.7 * 0.2
P(H|C) * P(1|H)
0.4 * 0.2
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
3
1
3
O_1
O_2
O_3



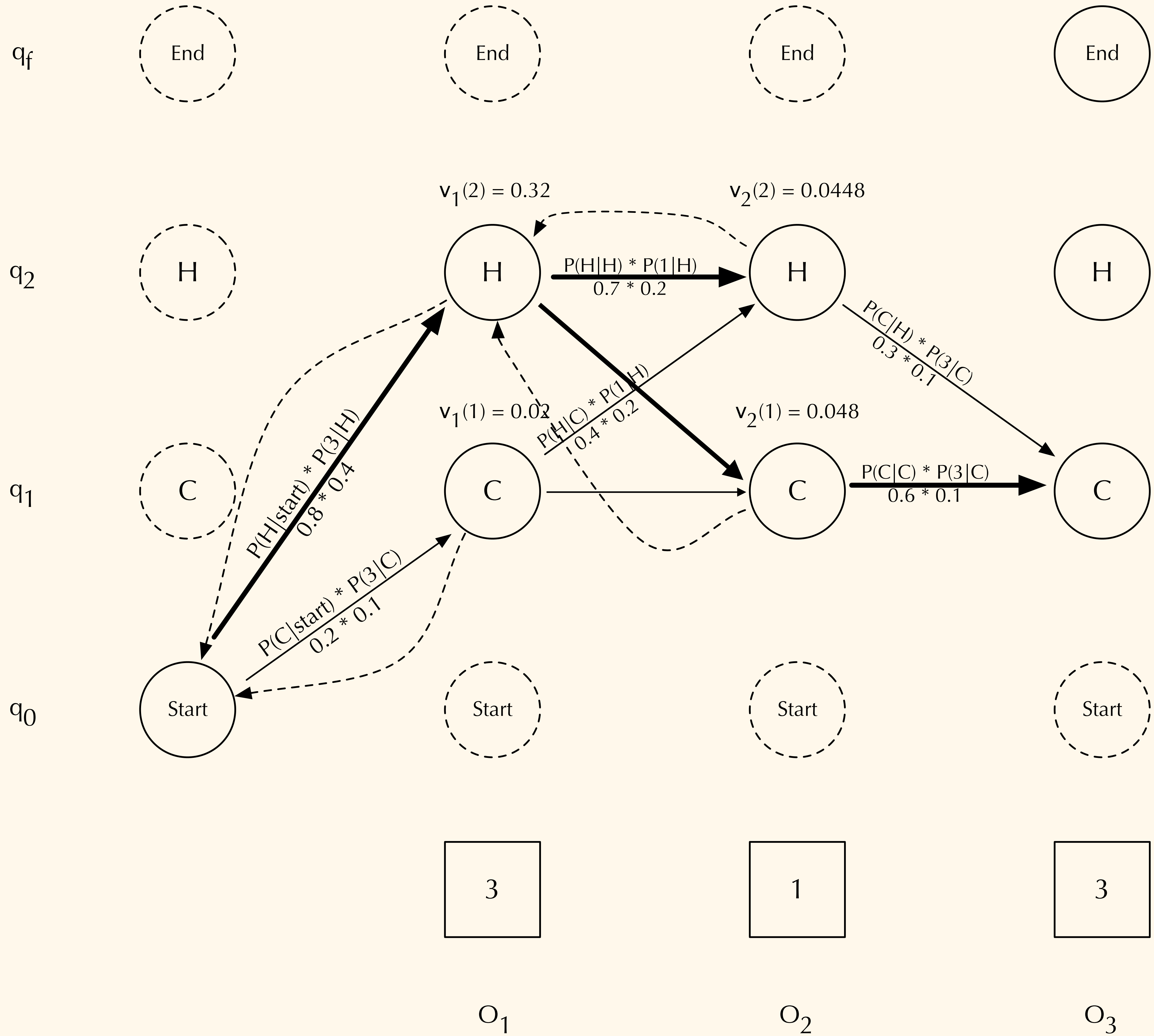
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$v_1(1) = 0.02$
$v_2(1) = 0.048$
P(H|C) * P(1|H)
0.4 * 0.2
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



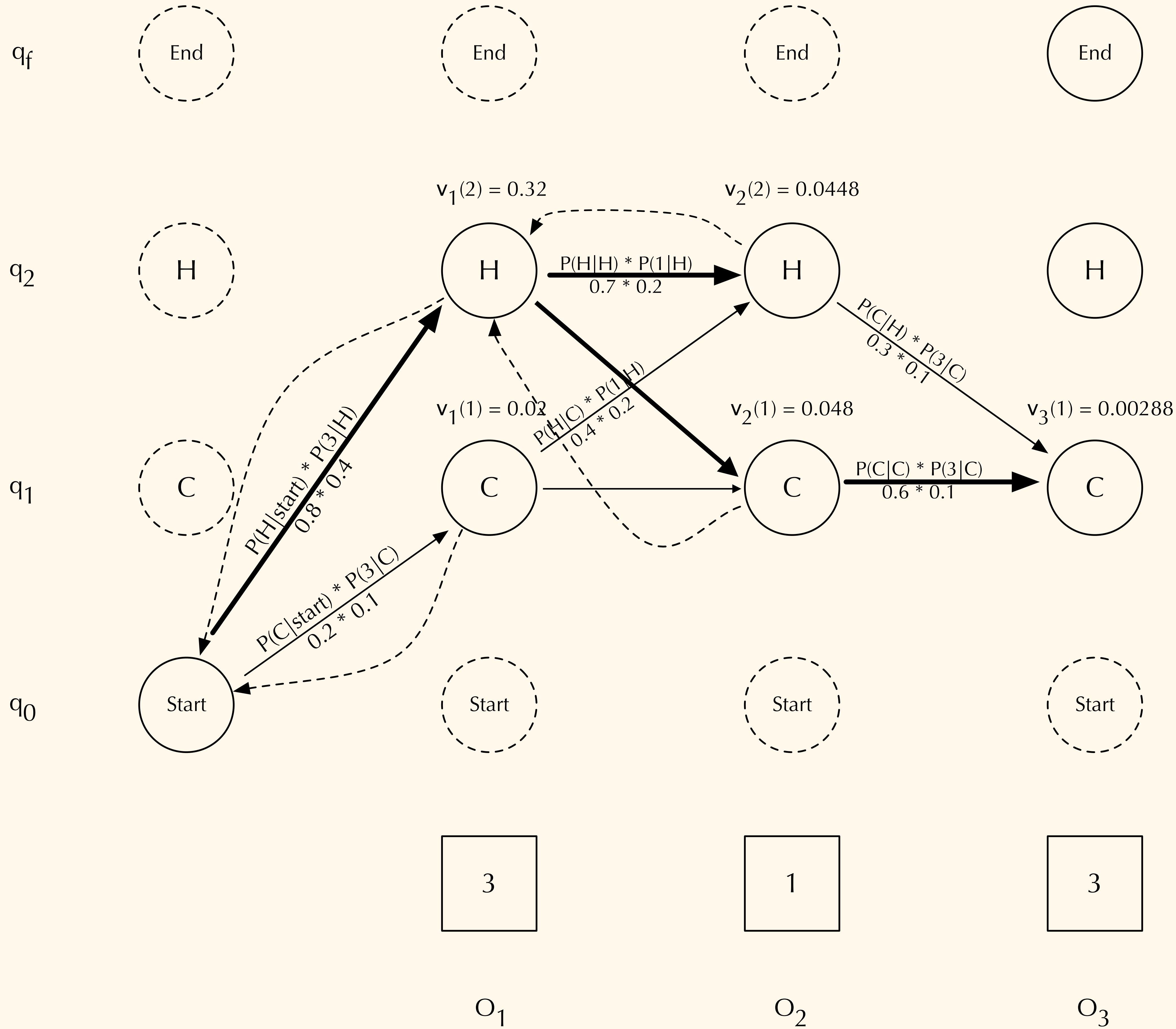
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$v_1(1) = 0.02$
$v_2(1) = 0.048$
P(H|C) * P(1|H)
0.4 * 0.2
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



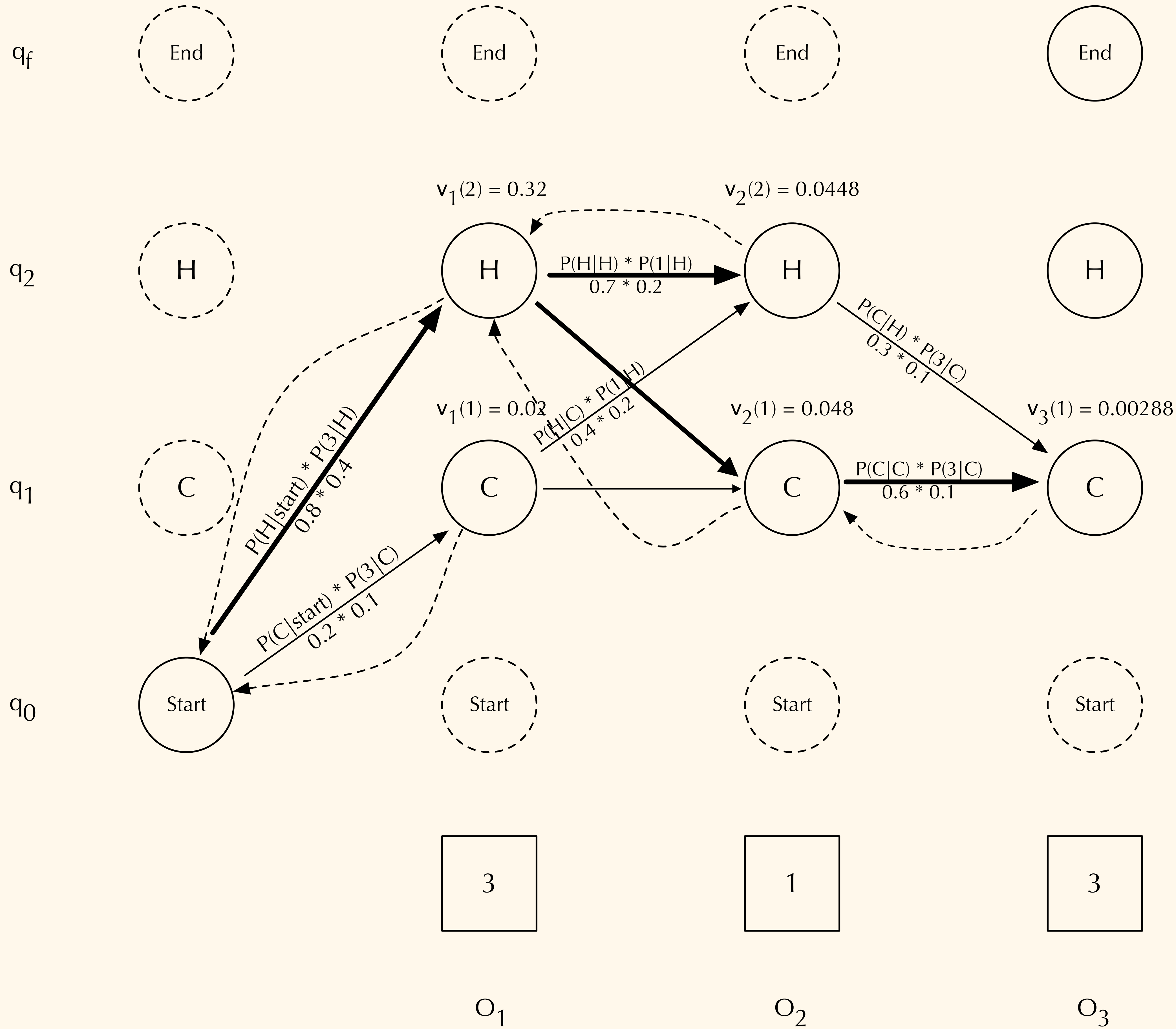
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$v_1(1) = 0.02$
$v_2(1) = 0.048$
P(H|C) * P(1|H)
0.4 * 0.2
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



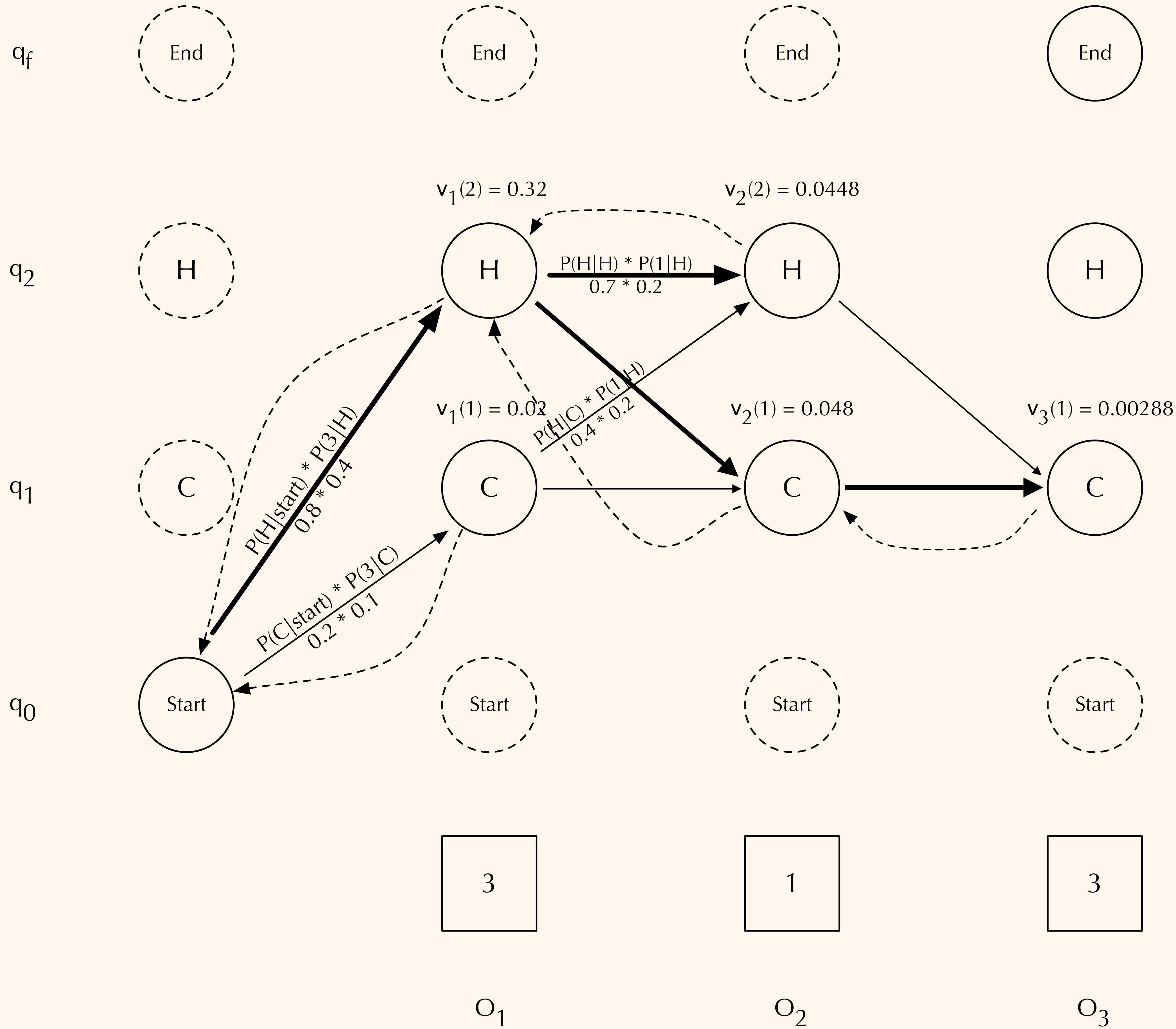
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(C|H) * P(3|C)
0.3 * 0.1
$v_1(1) = 0.02$
$v_2(1) = 0.048$
P(H|C) * P(1|H)
0.4 * 0.2
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|C) * P(3|C)
0.6 * 0.1
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



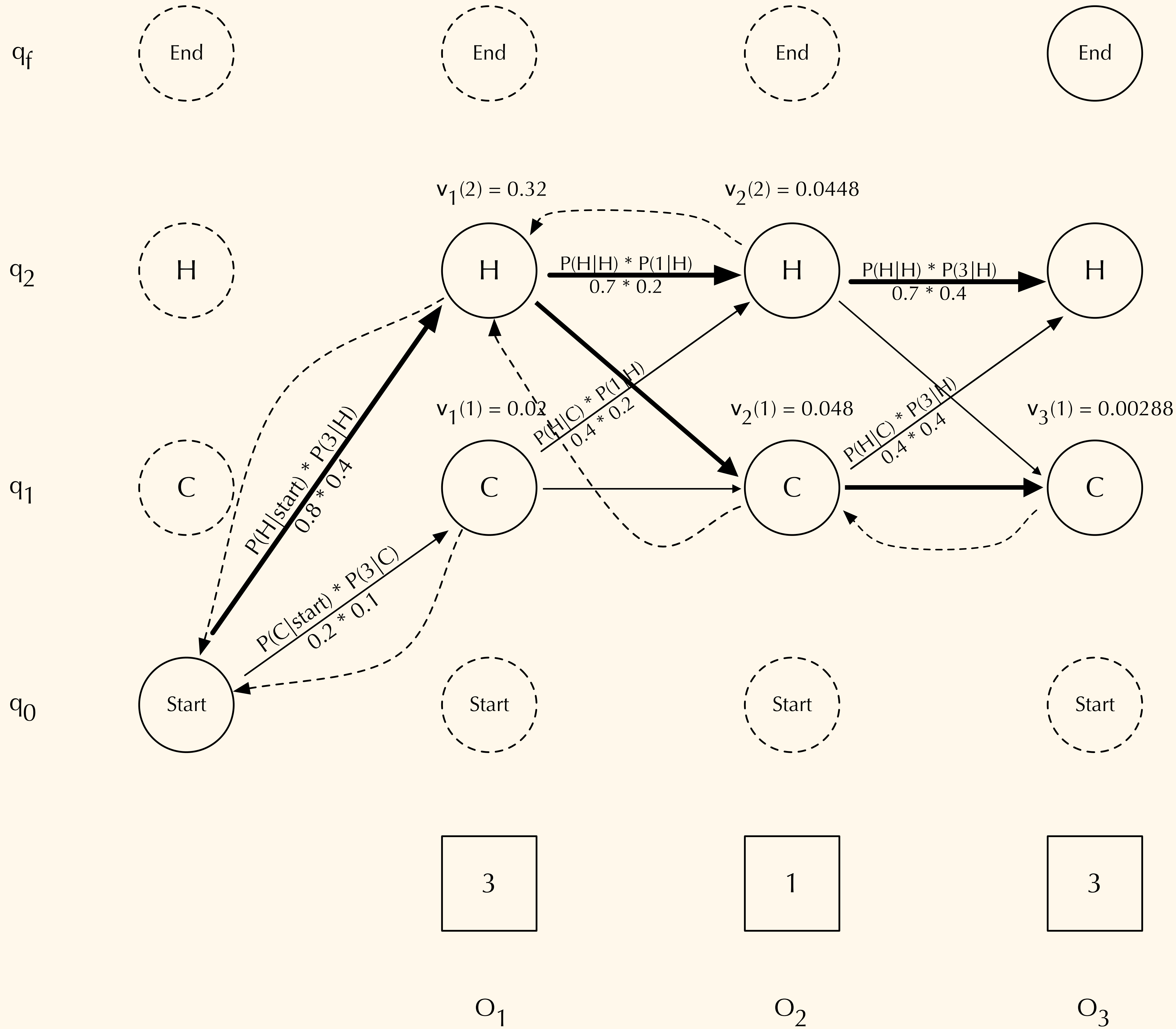
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(C|H) * P(3|C)
0.3 * 0.1
$v_1(1) = 0.02$
P(H|C) * P(1|H)
0.4 * 0.2
$v_2(1) = 0.048$
$v_3(1) = 0.00288$
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|C) * P(3|C)
0.6 * 0.1
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



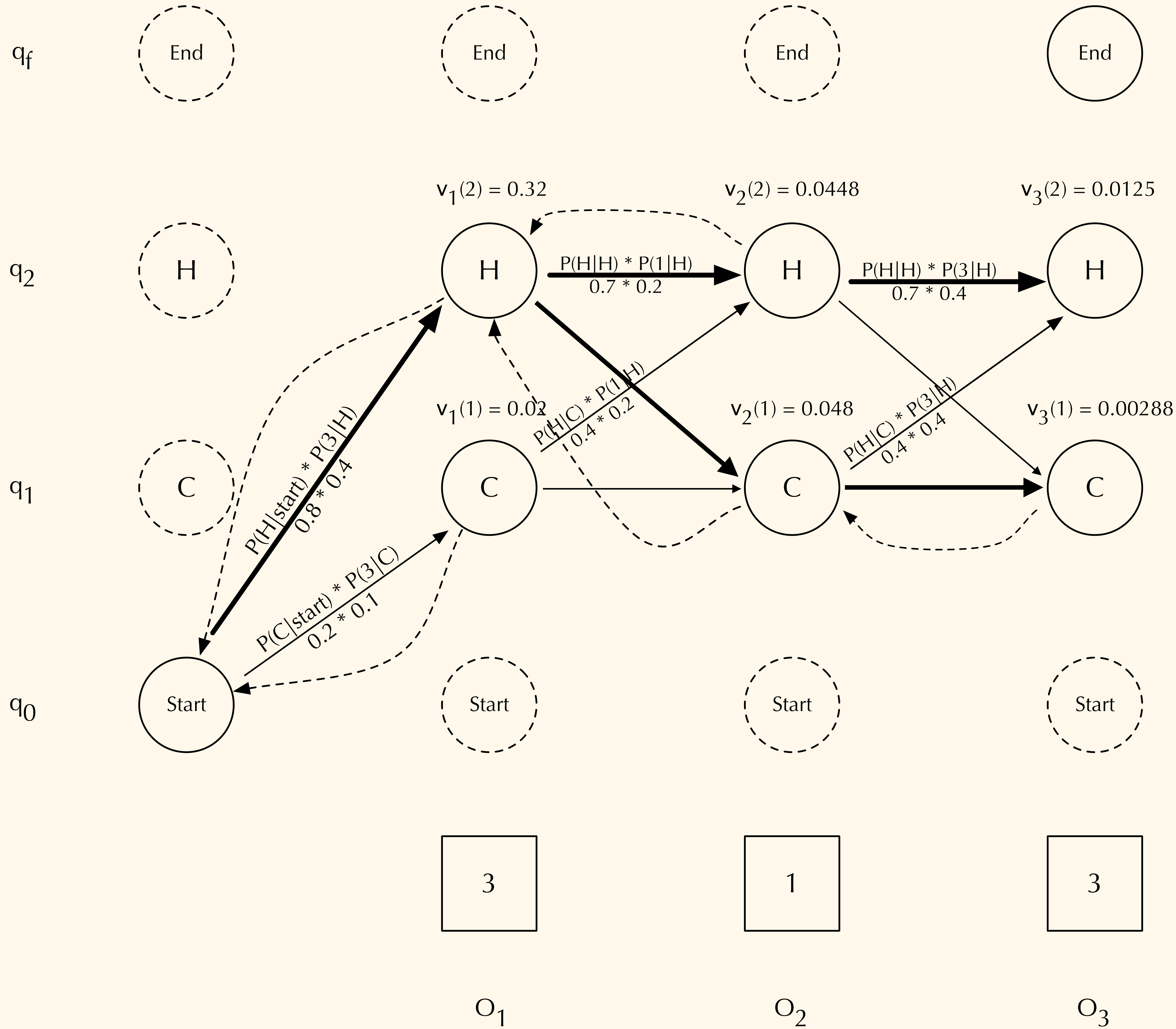
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
v_1(2) = 0.32
v_2(2) = 0.0448
v_1(1) = 0.02
v_2(1) = 0.048
v_3(1) = 0.00288
P(H|H) * P(1|H)
0.7 * 0.2
P(C|H) * P(3|C)
0.3 * 0.1
P(H|C) * P(1|H)
0.4 * 0.2
P(C|C) * P(3|C)
0.6 * 0.1
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
3
1
3
O_1
O_2
O_3



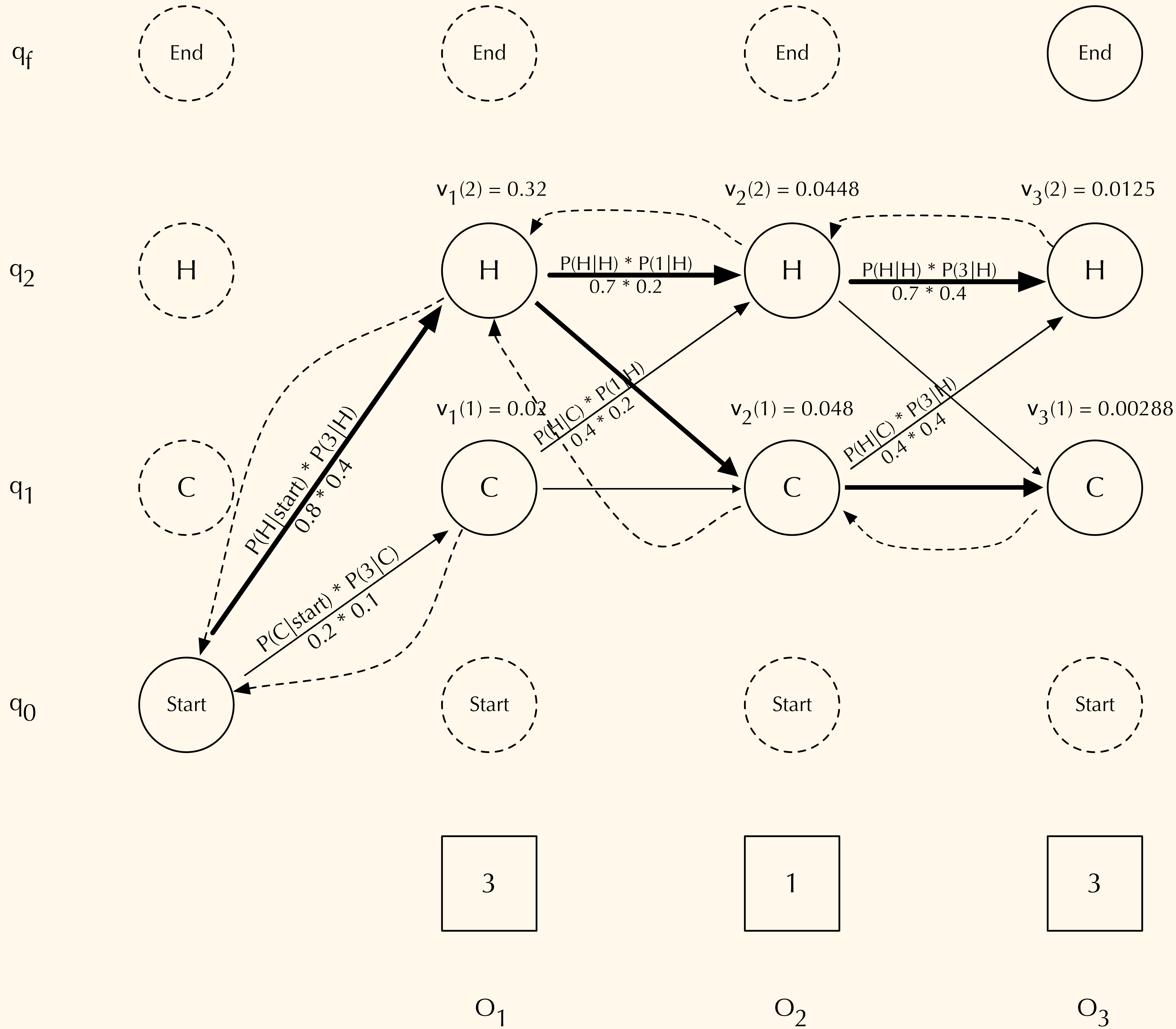
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
$v_1(1) = 0.02$
$v_2(1) = 0.048$
$v_3(1) = 0.00288$
P(H|C) * P(1|H)
0.4 * 0.2
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



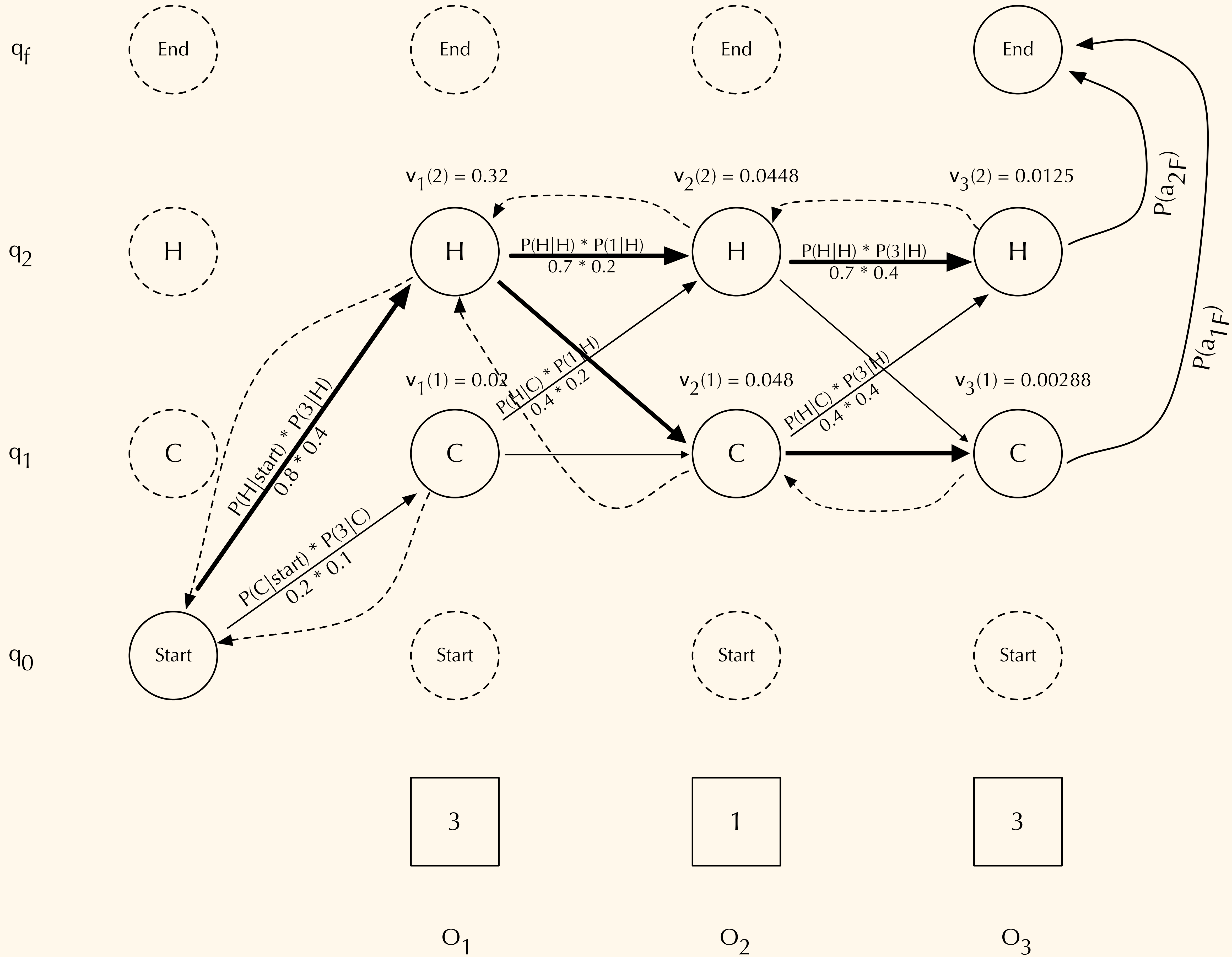
qf
q2
q1
q0
End
End
End
End
v1(2) = 0.32
v2(2) = 0.0448
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
v1(1) = 0.02
v2(1) = 0.048
v3(1) = 0.00288
P(H|start) * P(3|H)
0.8 * 0.4
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O1
O2
O3



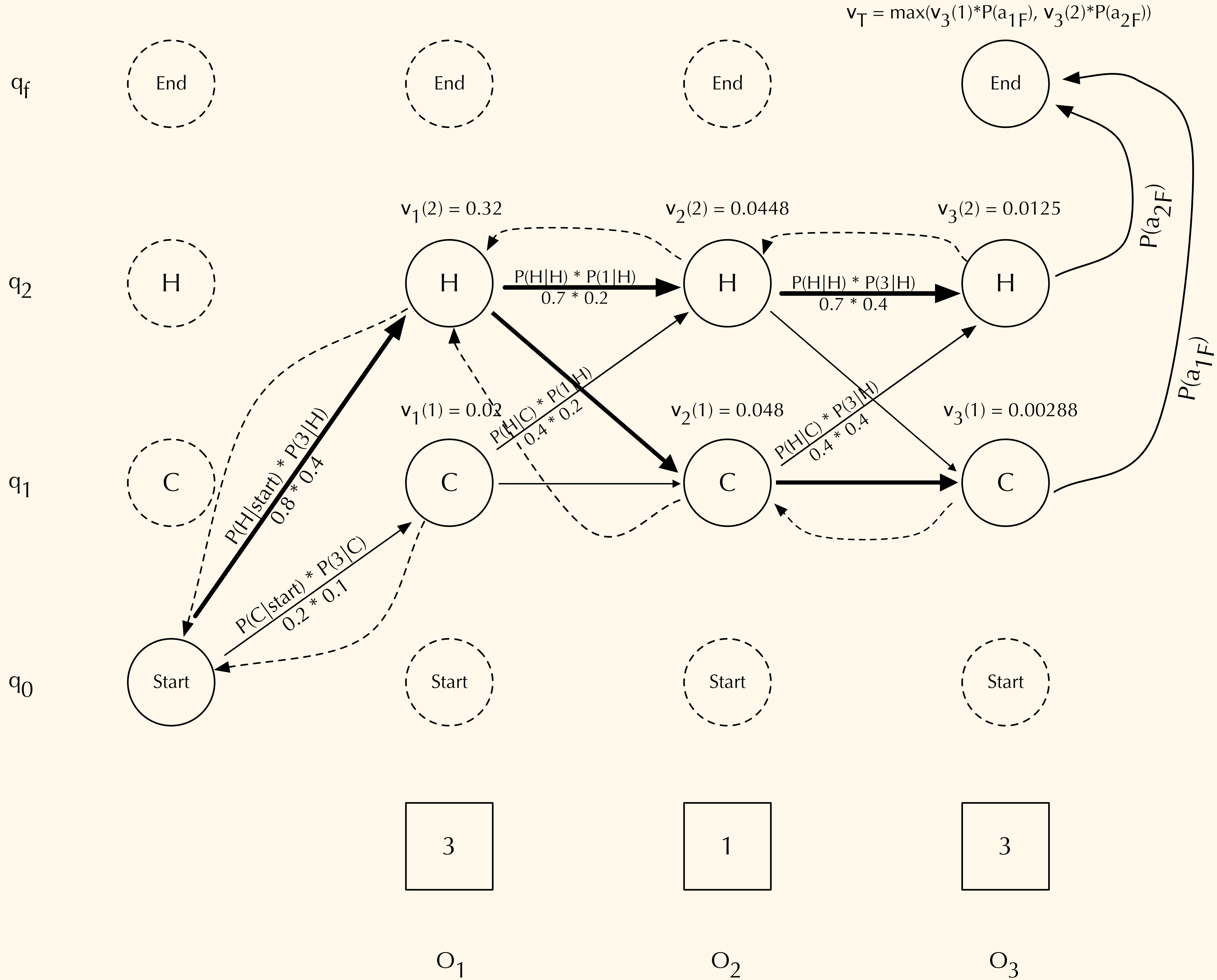
q_f
q_2
q_1
q_0
End
End
End
End
v_1(2) = 0.32
v_2(2) = 0.0448
v_3(2) = 0.0125
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
v_1(1) = 0.02
v_2(1) = 0.048
v_3(1) = 0.00288
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
P(H|start) * P(3|H)
0.8 * 0.4
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



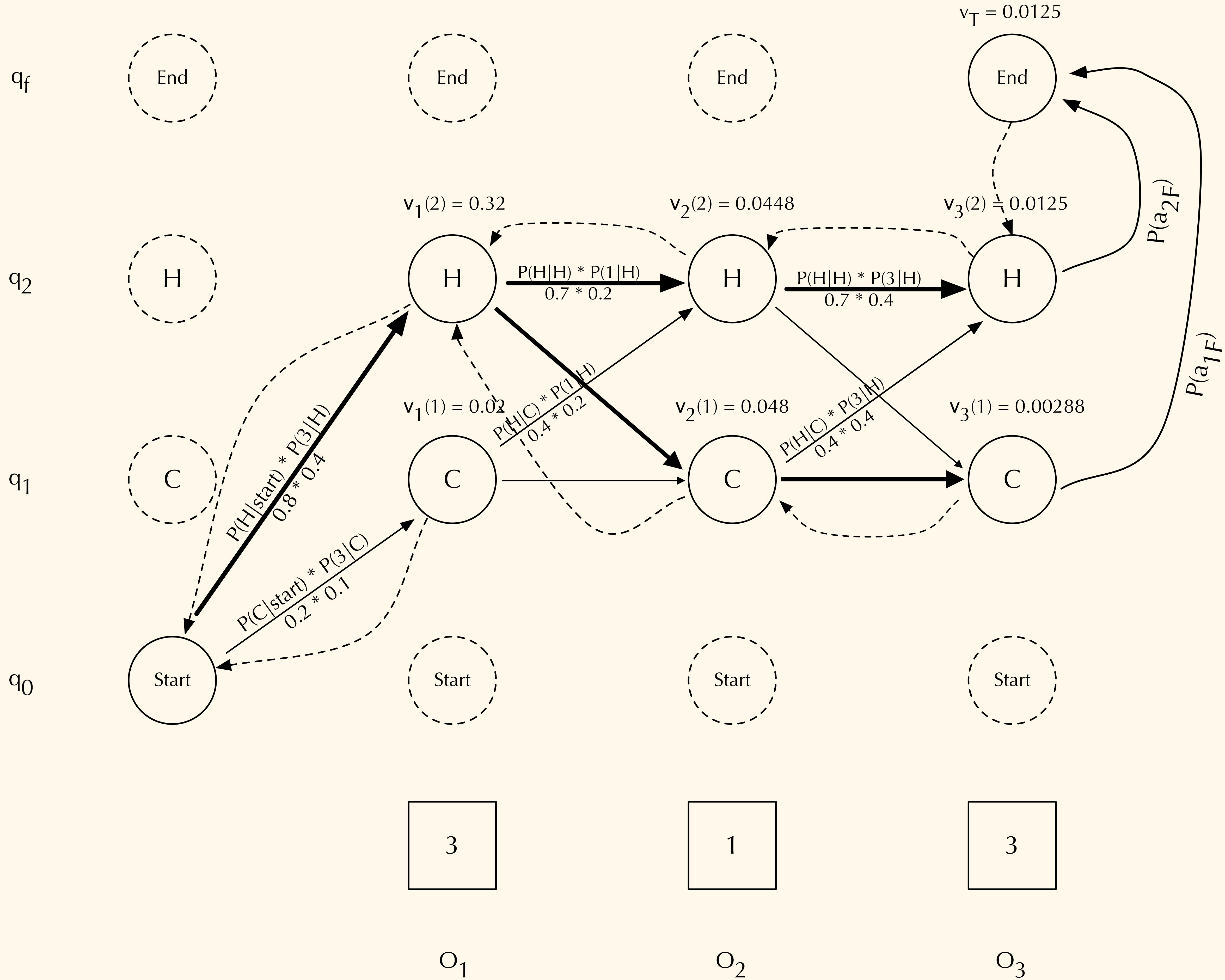
q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
$v_3(2) = 0.0125$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
$v_1(1) = 0.02$
$v_2(1) = 0.048$
$v_3(1) = 0.00288$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



q_f
q_2
q_1
q_0
End
End
End
End
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
$v_3(2) = 0.0125$
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
$P(a_{2F})$
$P(a_{1F})$
$v_1(1) = 0.02$
$v_2(1) = 0.048$
$v_3(1) = 0.00288$
C
C
C
C
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



$v_T = \max(v_3(1)*P(a_{1F}), v_3(2)*P(a_{2F}))$
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$v_1(2) = 0.32$
$v_2(2) = 0.0448$
$v_3(2) = 0.0125$
$v_1(1) = 0.02$
$v_2(1) = 0.048$
$v_3(1) = 0.00288$
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
P(H|start) * P(3|H)
0.8 * 0.4
P(C|start) * P(3|C)
0.2 * 0.1
$P(a_{2F})$
$P(a_{1F})$
3
1
3
o_1
o_2
o_3



v_T = 0.0125
q_f
End
End
End
End
$v_1(2)$ = 0.32
$v_2(2)$ = 0.0448
$v_3(2)$ = 0.0125
$P(a_{2F})$
q_2
H
H
H
H
P(H|H) * P(1|H)
0.7 * 0.2
P(H|H) * P(3|H)
0.7 * 0.4
$P(a_{1F})$
$v_1(1)$ = 0.02
$v_2(1)$ = 0.048
$v_3(1)$ = 0.00288
P(H|C) * P(1|H)
0.4 * 0.2
P(H|C) * P(3|H)
0.4 * 0.4
P(H|start) * P(3|H)
0.8 * 0.4
q_1
C
C
C
C
P(C|start) * P(3|C)
0.2 * 0.1
q_0
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3

There are three fundamental kinds of questions that we can ask with an HMM:

1. *Likelihood:* Given a sequence of states, what is the most likely observed sequence? *or* How likely is a given observation sequence?

2. ***Decoding:*** **Given an observation sequence and a fully-specified HMM, what is the most likely sequence of states to have produced that observation?**

3. *Learning:* Given an observation sequence and a set of states, what are the likely transition and emission probabilities (A and B)?

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Lloyd Welch

Recall from before that the *forward probability* $\alpha_t(i)$ is the probability of ending up in state *i* given observations $O_{1:t}$.

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A related property is the *backward probability* $\beta_t(i)$, which represents the probability of seeing observations $O_{t:T}$, given that we are currently in state *i* at time *t*.

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This is calculated using the *backward algorithm*, which is very similar to the forward algorithm (but in reverse!).

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Probability of finishing (i.e., reaching end state) the observed sequence from state *i*.

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$$\beta_T(i) = a_{i,F}$$

Probability of finishing (i.e., reaching end state) the observed sequence from state *i*.

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

This is calculated using the *backward algorithm*, which is very similar to the forward algorithm (but in reverse!)

$$\beta_T(i) = a_{i,F}$$

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$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

Sum of the backwards probabilities of the different paths through the model that could happen from state *i* and time *t*.

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$$\beta_T(i) = a_{i,F}$$

Probability of finishing (i.e., reaching end state) the observed sequence from state *i*.

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

Sum of the backwards probabilities of the different paths through the model that could happen from state *i* and time *t*.

$$P(O|\lambda) = \alpha_T(q_F) = \beta_1(0) = \sum_{j=1}^{N} a_{0j} b_j(o_1) \beta_1(j)$$

This is calculated using the *backward algorithm*, which is very similar to the forward algorithm (but in reverse!)

$$\beta_T(i) = a_{i,F}$$

Probability of finishing (i.e., reaching end state) the observed sequence from state *i*.

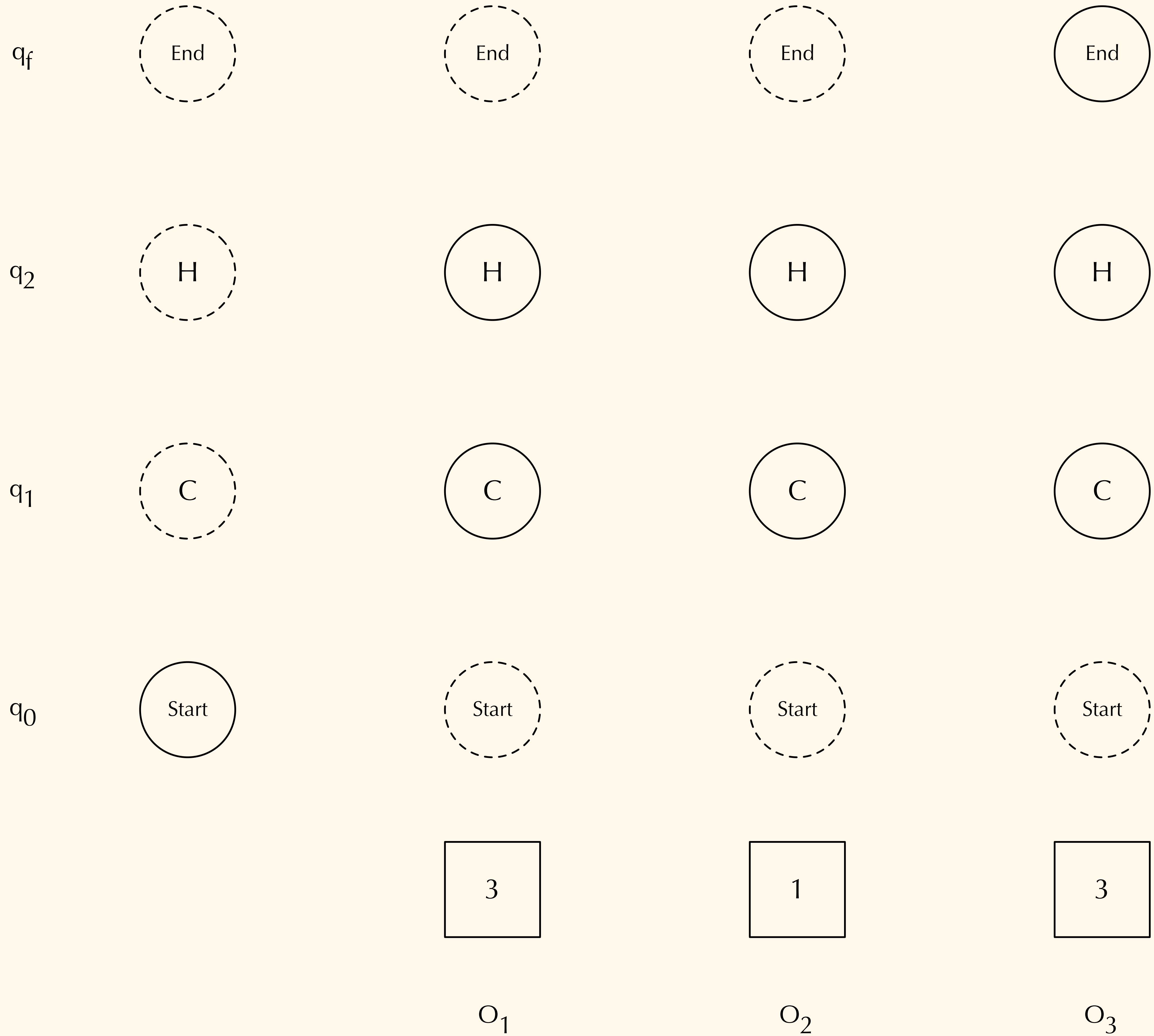
$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$

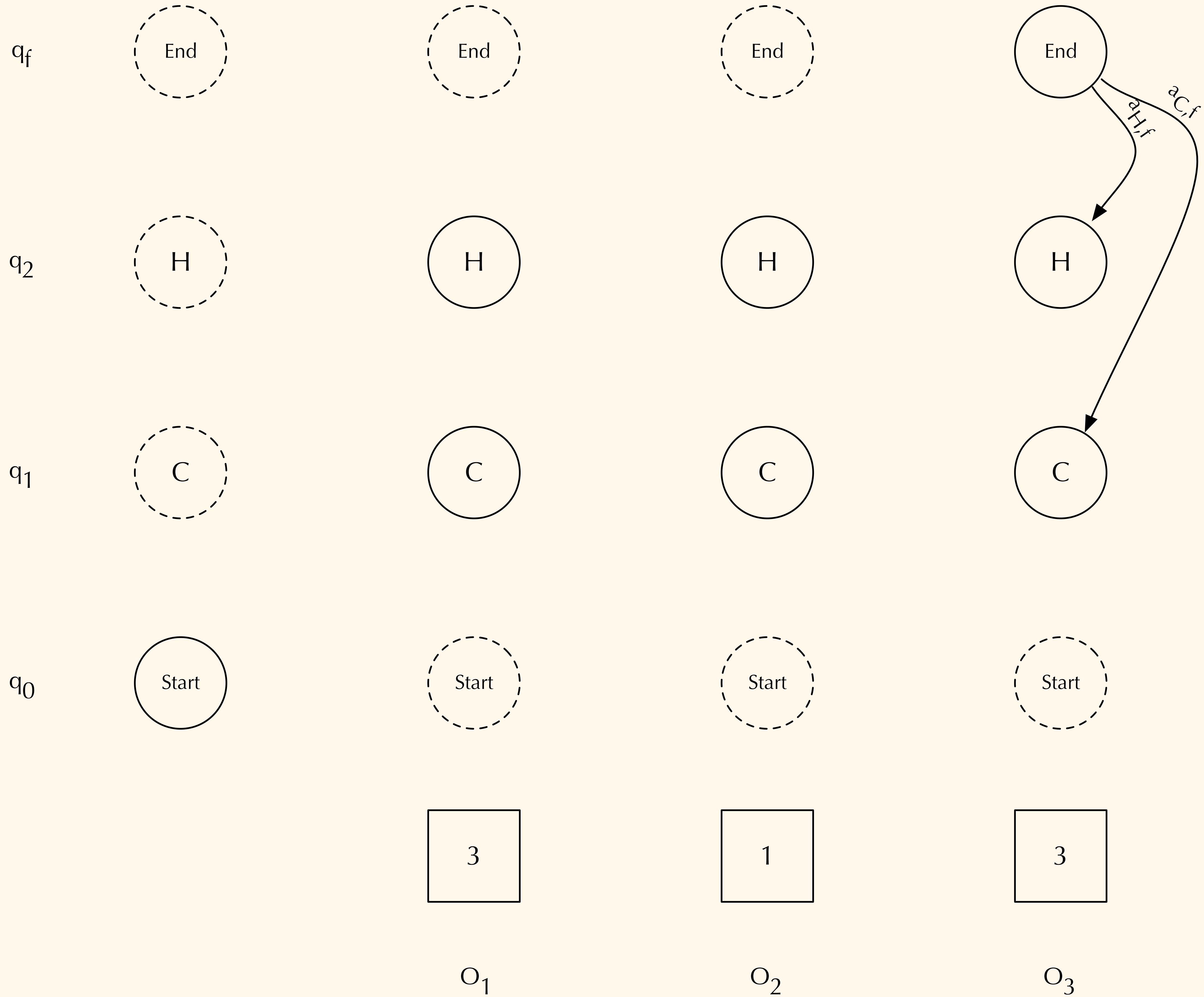
Sum of the backwards probabilities of the different paths through the model that could happen from state *i* and time *t*.

$$P(O|\lambda) = \alpha_T(q_F) = \beta_1(0) = \sum_{j=1}^{N} a_{0j} b_j(o_1) \beta_1(j)$$

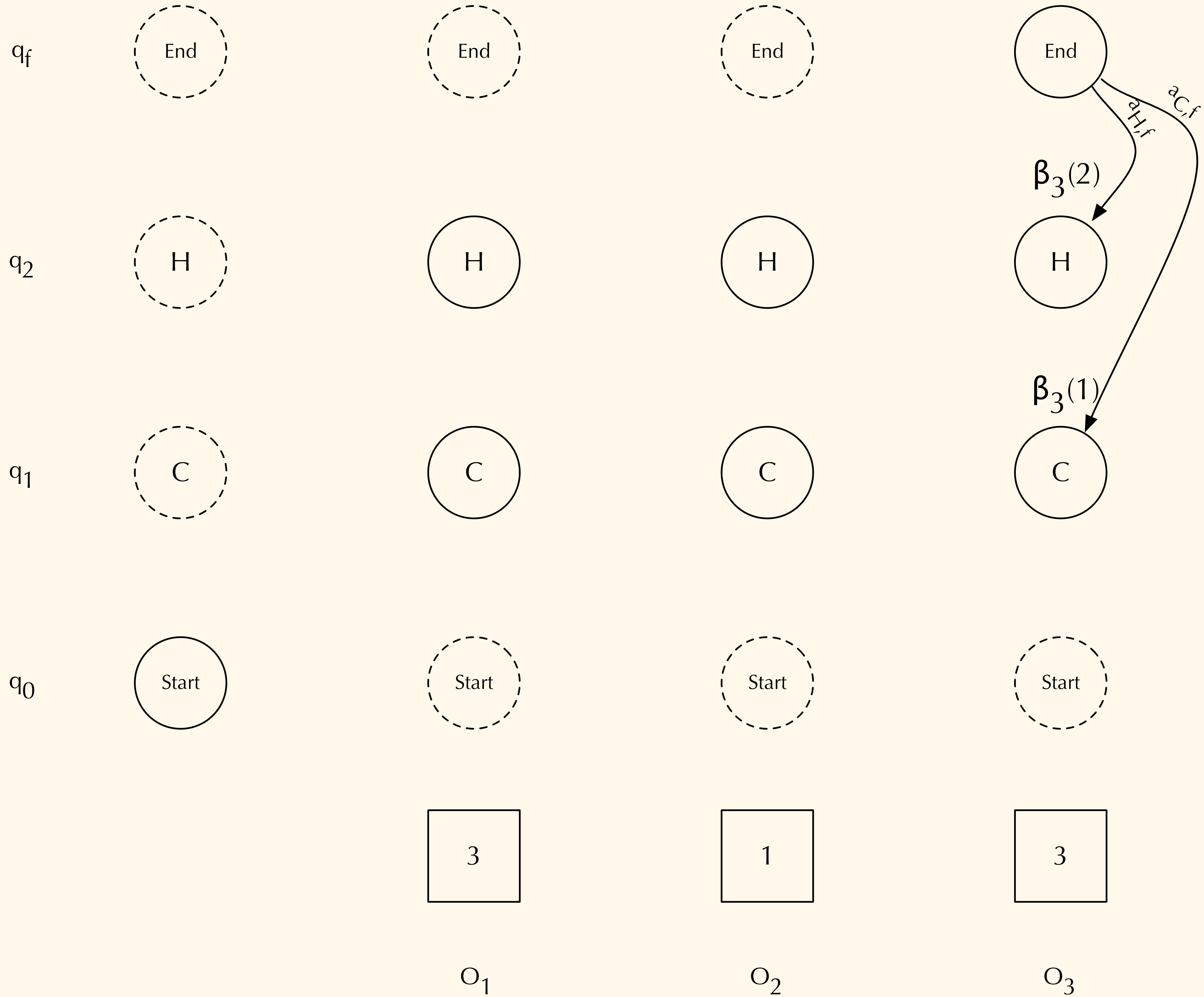
Final forward probability of observation given model.

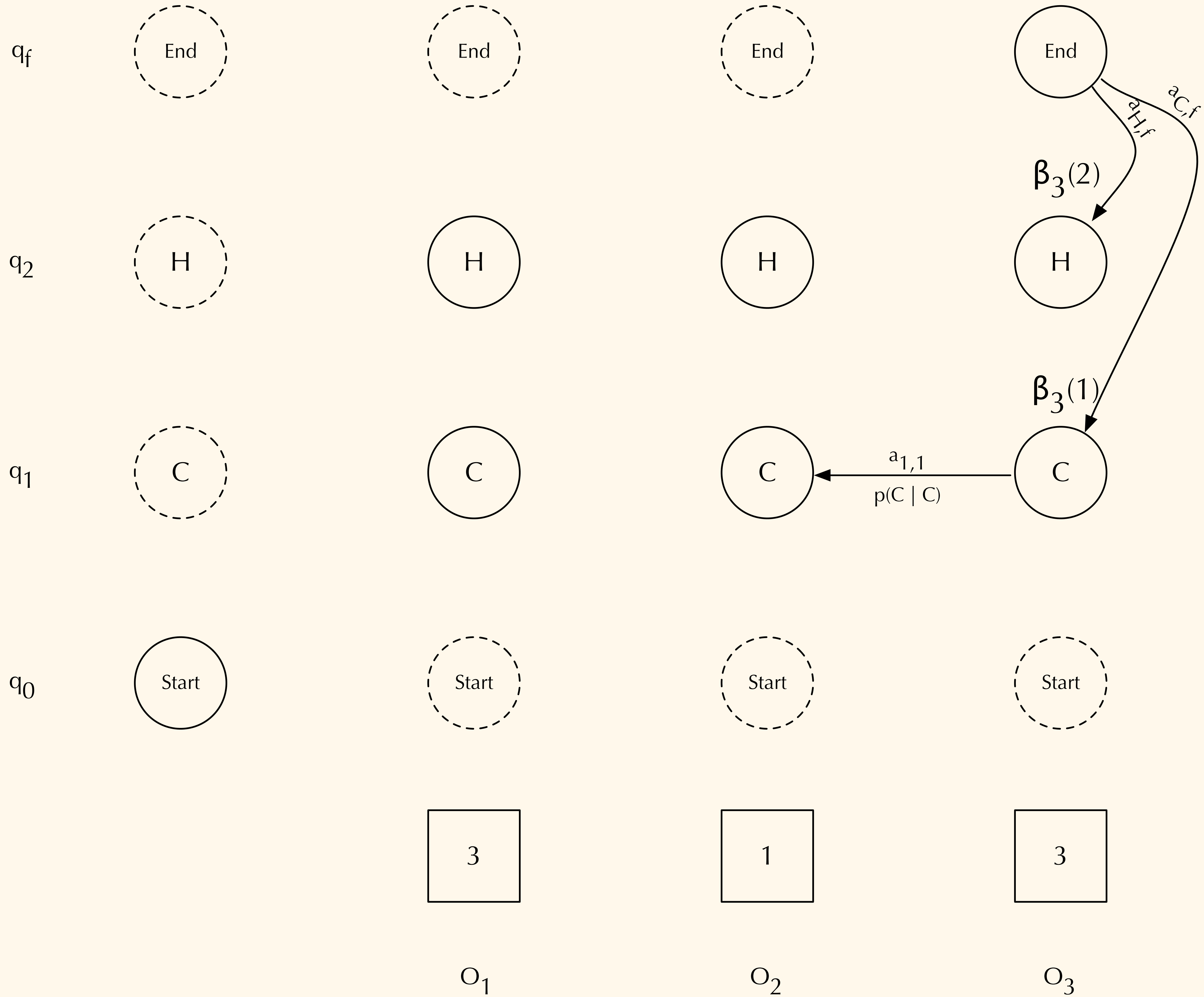
Of course, we compute all of this using the same dynamic programming approach we've already seen:


q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3

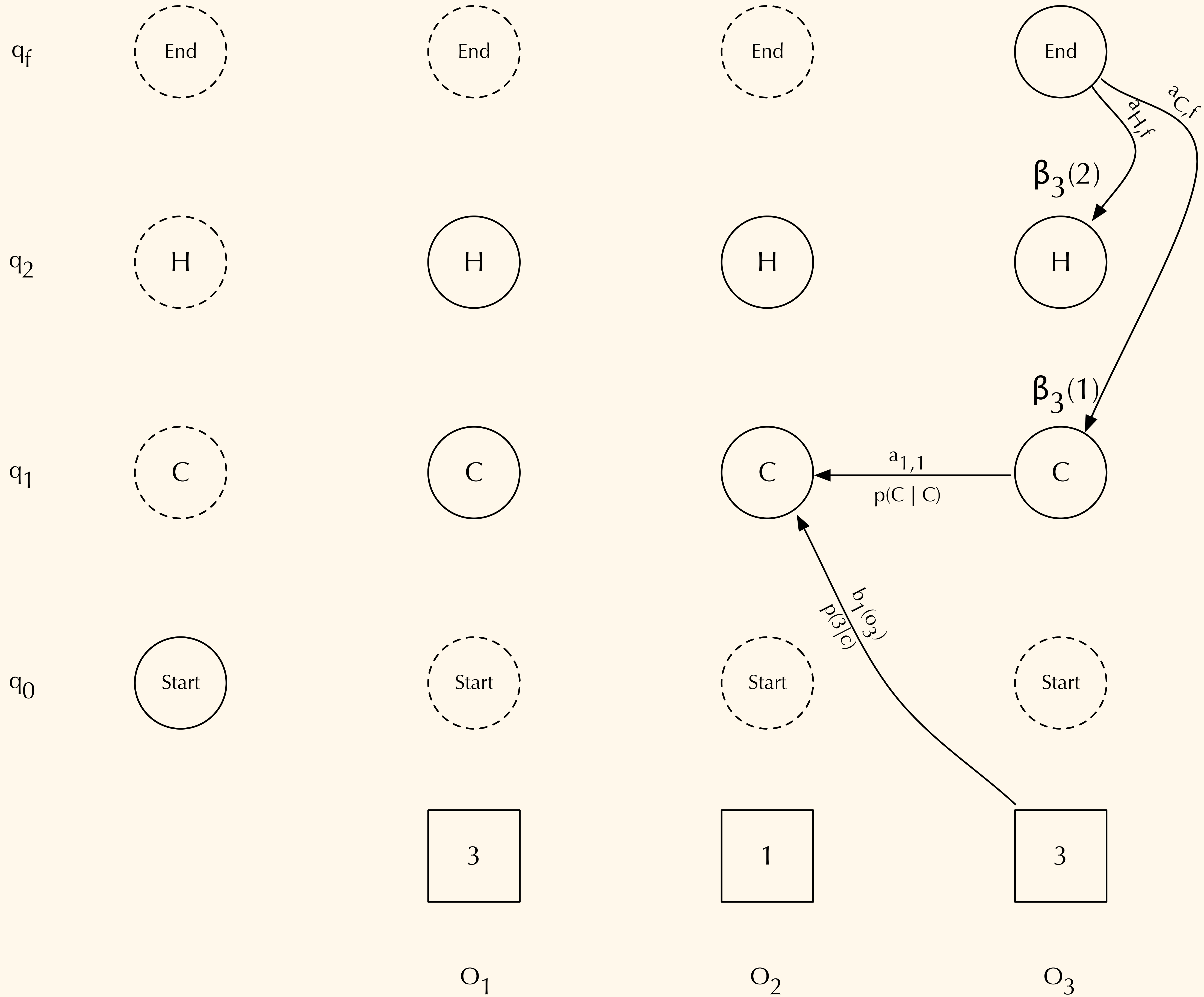


q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$a_{H,f}$
$a_{C,f}$
3
1
3
o_1
o_2
o_3

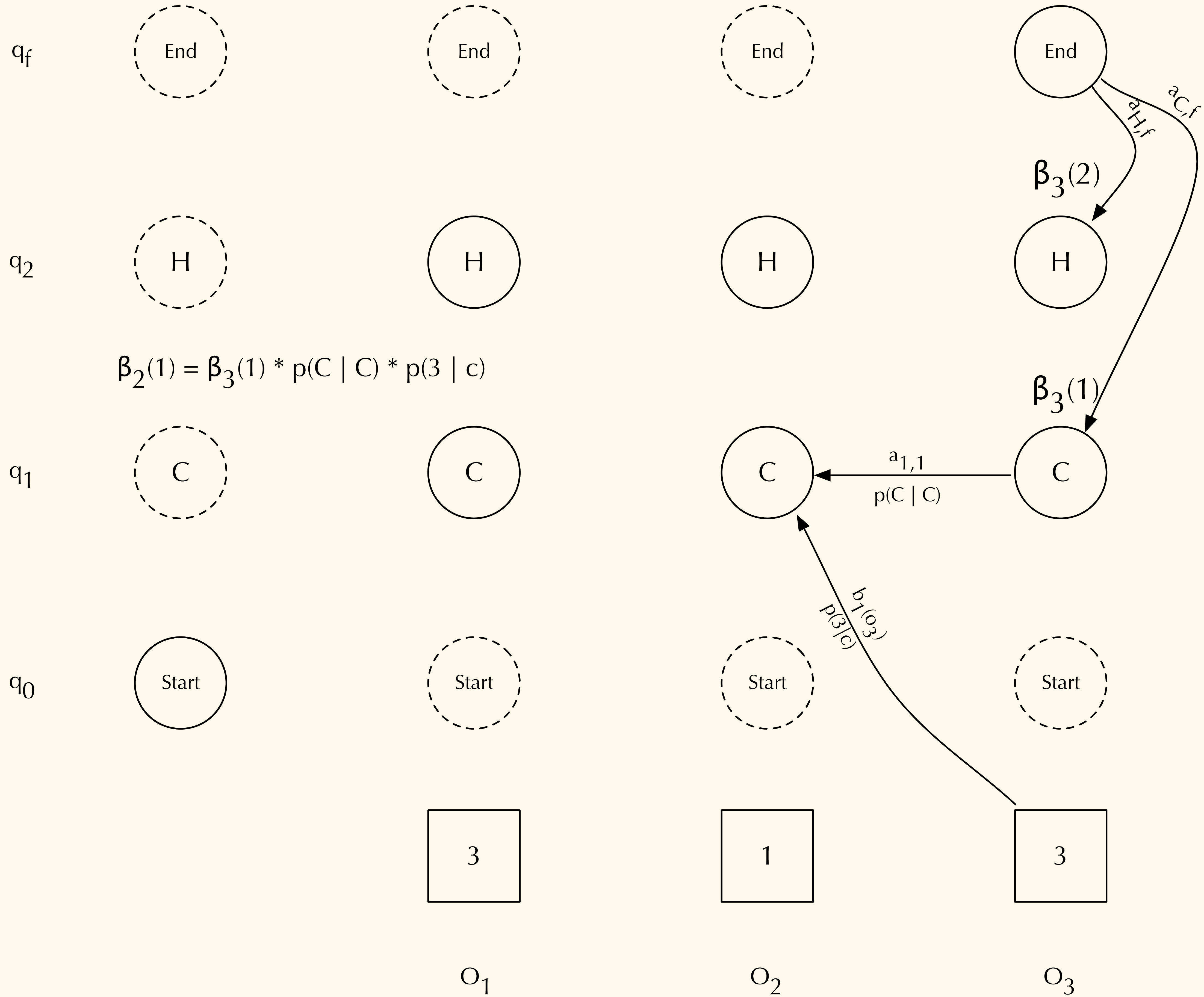

q_f
q_2
q_1
q_0
End
End
End
End
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
H
H
H
H
$\beta_3(1)$
C
C
C
C
Start
Start
Start
Start
3
1
3
o_1
o_2
o_3



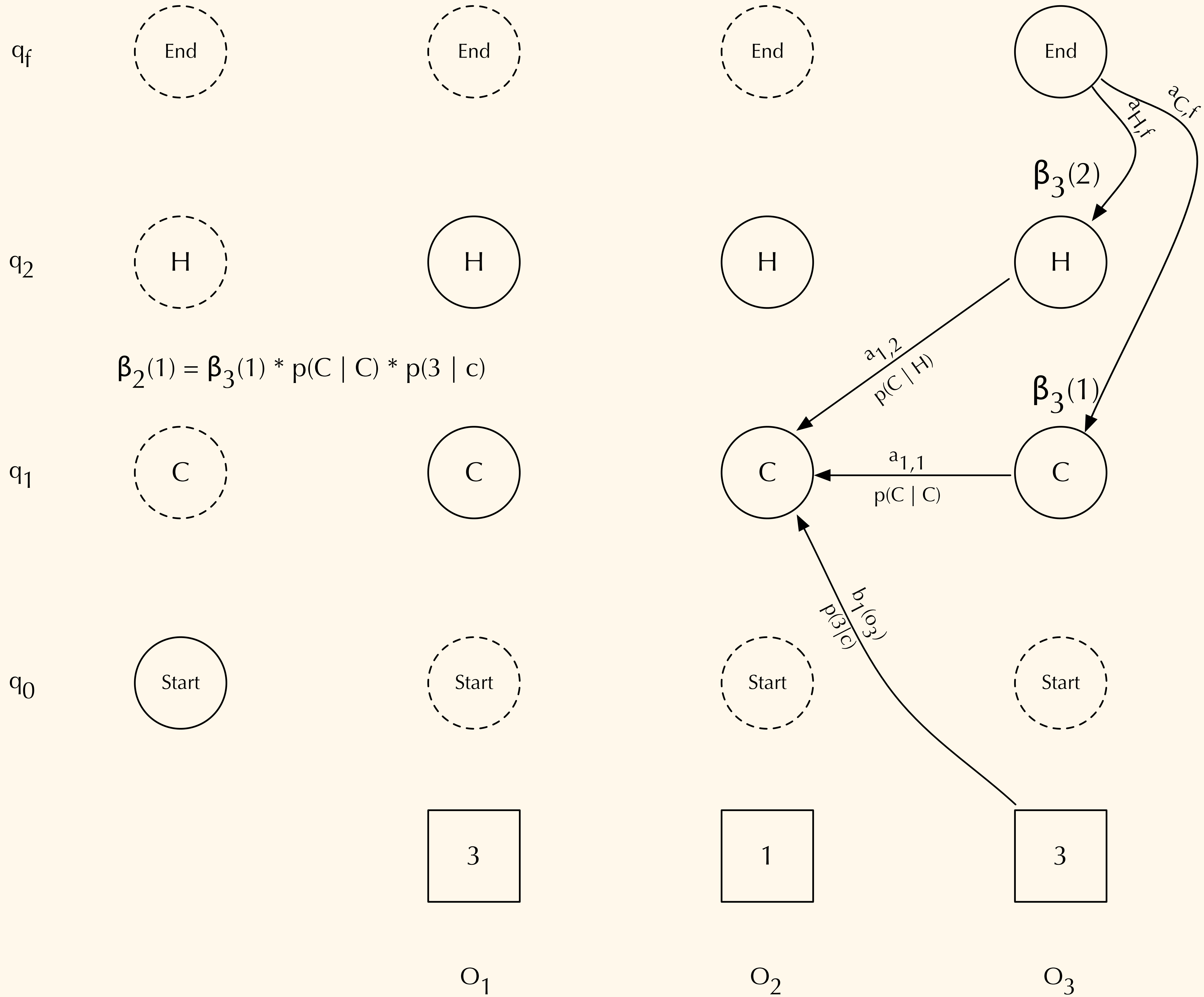
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
$\beta_3(1)$
$a_{1,1}$
p(C | C)
3
1
3
O_1
O_2
O_3



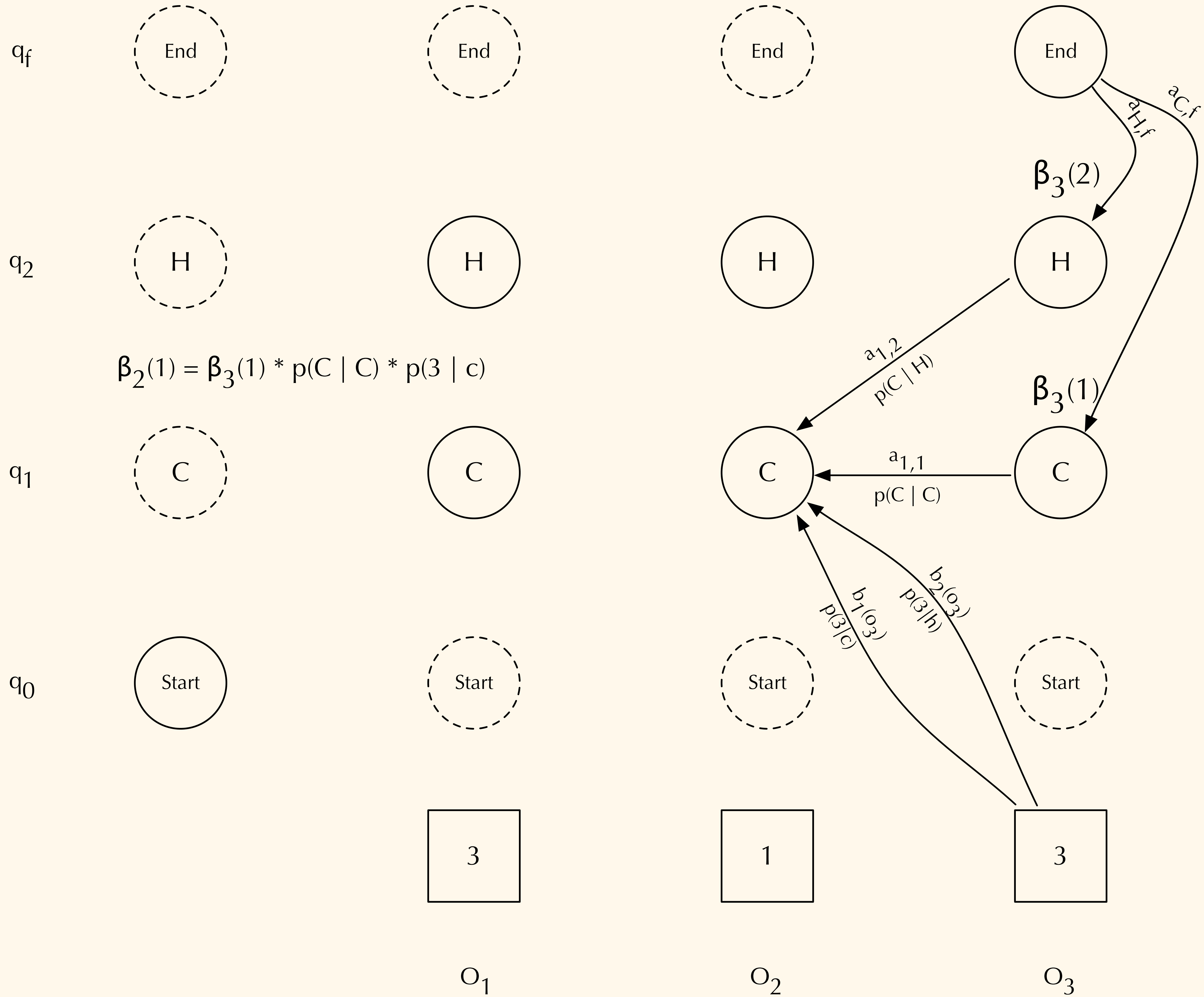
q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
$\beta_3(1)$
$a_{1,1}$
p(C | C)
$b_1(o_3)$
p(3|c)
3
1
3
O_1
O_2
O_3

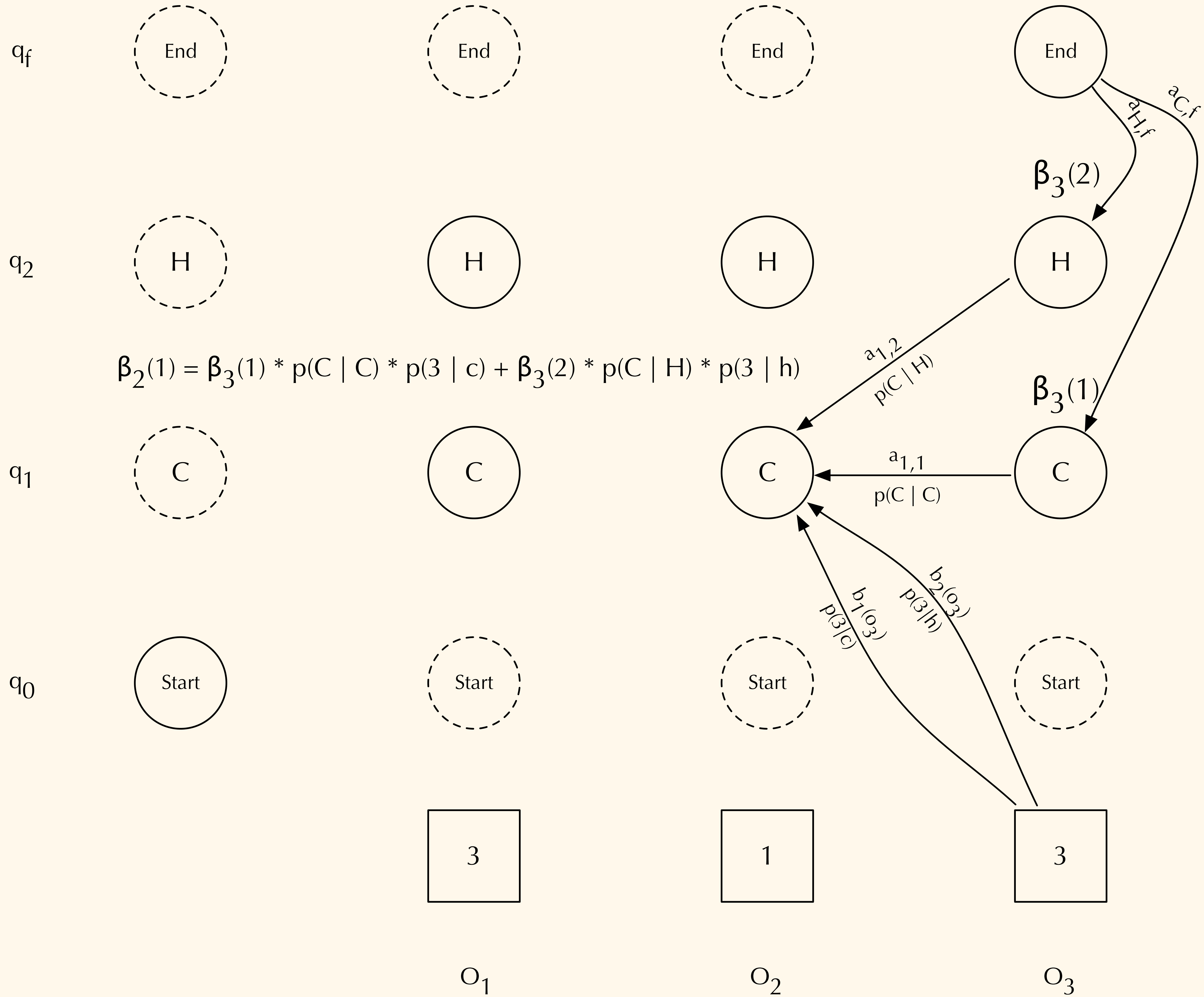


q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
$\beta_3(1)$
$\beta_2(1) = \beta_3(1) * p(C \mid C) * p(3 \mid c)$
$a_{1,1}$
p(C | C)
$b_1(o_3)$
p(3|c)
3
1
3
O_1
O_2
O_3



q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
$\beta_3(2)$
$a_{H,f}$
$a_{C,f}$
$\beta_2(1) = \beta_3(1) * p(C \mid C) * p(3 \mid c)$
$a_{1,2}$
$p(C \mid H)$
$\beta_3(1)$
C
C
C
C
$a_{1,1}$
$p(C \mid C)$
$b_1(o_3)$
$p(3|c)$
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3


q_f
End
End
End
End
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
q_2
H
H
H
H
$\beta_2(1) = \beta_3(1) * p(C \mid C) * p(3 \mid c)$
$a_{1,2}$
p(C | H)
$\beta_3(1)$
q_1
C
C
C
C
$a_{1,1}$
p(C | C)
$b_1(o_3)$
p(3|c)
$b_2(o_3)$
p(3|h)
q_0
Start
Start
Start
Start
3
1
3
O_1
O_2
O_3



q_f
q_2
q_1
q_0
End
End
End
End
H
H
H
H
C
C
C
C
Start
Start
Start
Start
$a_{H,f}$
$a_{C,f}$
$\beta_3(2)$
$\beta_3(1)$
$\beta_2(1) = \beta_3(1) * p(C \mid C) * p(3 \mid c) + \beta_3(2) * p(C \mid H) * p(3 \mid h)$
$a_{1,2}$
$p(C \mid H)$
$a_{1,1}$
$p(C \mid C)$
$b_1(o_3)$
$p(3|c)$
$b_2(o_3)$
$p(3|h)$
3
1
3
O_1
O_2
O_3

Now, how to use these to estimate A and B?

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Baum-Welch is a variation on *Expectation-Maximization*...

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… as such, we start with a "guess" for A and B, and iteratively improve it.

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We begin by attempting to find:

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$$\hat{a}_{ij} = \frac{\text{expected \# transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

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We begin by attempting to find:

$$\hat{a}_{ij} = \frac{\text{expected \# transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

If we had an estimate of the probability of transition $i \rightarrow j$ occurring at each time t, we could sum them to get the total count for $i \rightarrow j$.

More formally: let $\xi_t(i,j)$ be the probability of being in state *i* at time *t* and in state *j* at time *t+1*.

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$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j | O, \lambda)$$

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We can't quite calculate this, but we *can* calculate something similar:

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We can't quite calculate this, but we *can* calculate something similar:

$$\tilde{\xi}_t(i,j) = P(q_t = i, q_{t+1} = j, O | \lambda)$$

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We have all the pieces we need to get this:

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We have all the pieces we need to get this:

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Forward probability of
observations up to this arc

We have all the pieces we need to get this:

$$\tilde{\xi}_t(i,j) = \underbrace{\alpha_t(i)}_{\text{Forward probability of observations up to this arc}} \underbrace{a_{ij}}_{\text{Transition probability between states } i \text{ and } j} b_j(o_{t+1})\beta_{t+1}(j)$$

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Emission probability of the next symbol

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Transition probability between states *i* and *j*

Emission probability of the next symbol

Backward probability after this arc

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So, the final equation is:

$$\xi_t(i,j) = \frac{\alpha_t(i)a_{ij}b_j(o_{t+1})\beta_{t+1}(j)}{\alpha_T(N)}$$

Remember:

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"The probability of going from state *i* to state *j* at time *t*." (given a current estimate of the model).

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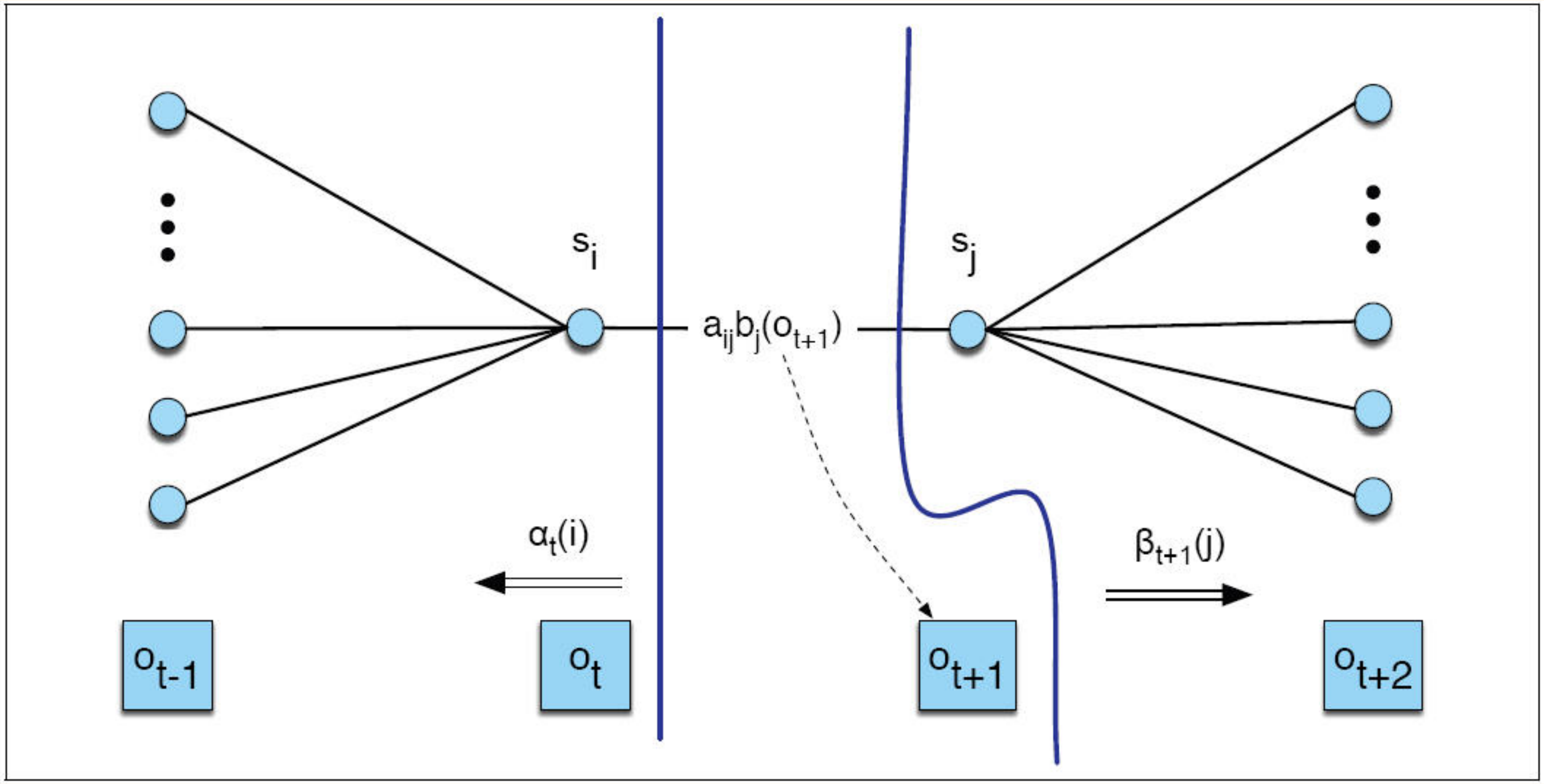
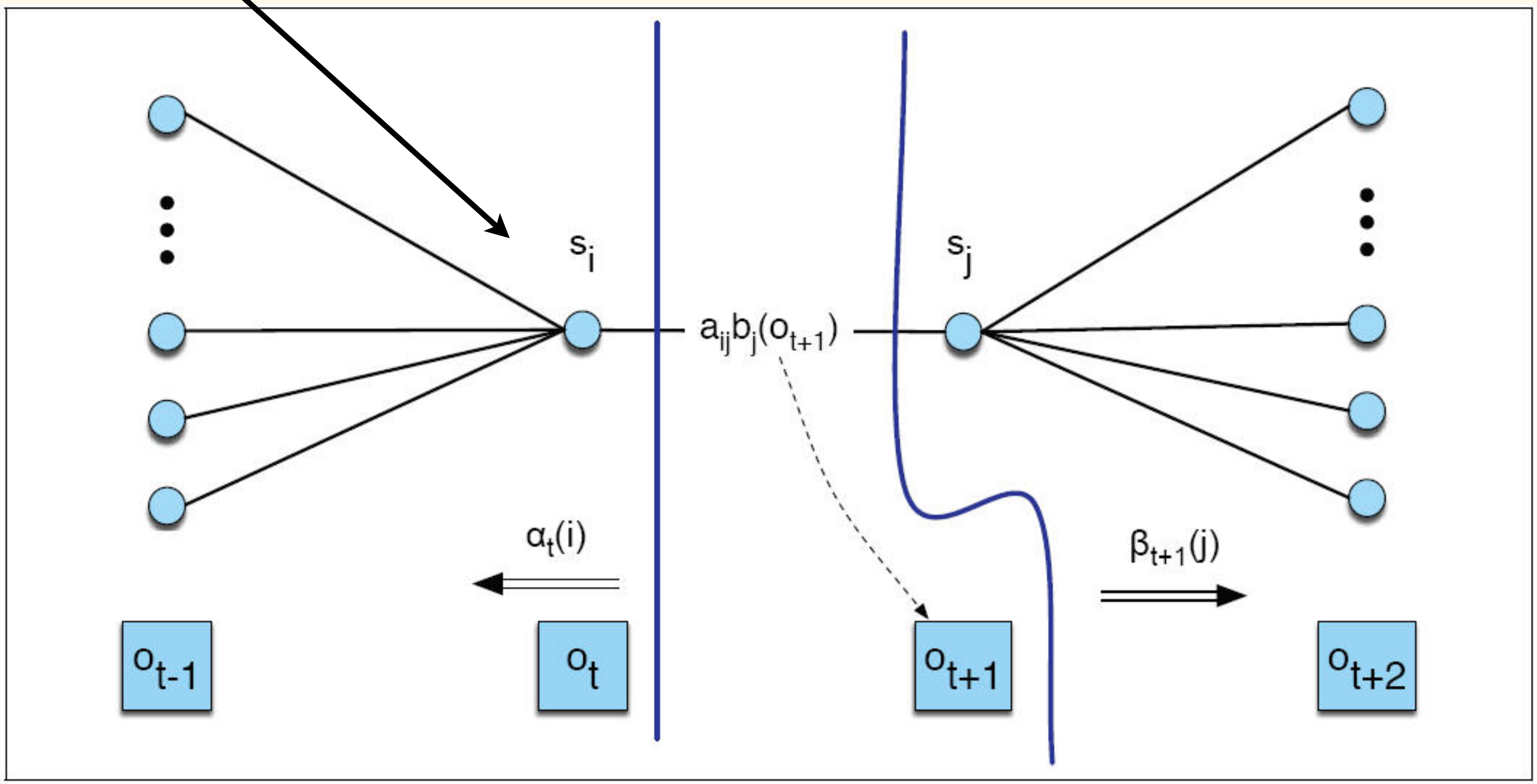
Which looks a lot like:

$$\hat{a}_{ij} = \frac{\text{expected } \# \text{ transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$



s_i
s_j
a_ij b_j(o_t+1)
α_t(i)
β_t+1(j)
o_t-1
o_t
o_t+1
o_t+2

All of the ways the model could have gotten into state *i* at time *t*...



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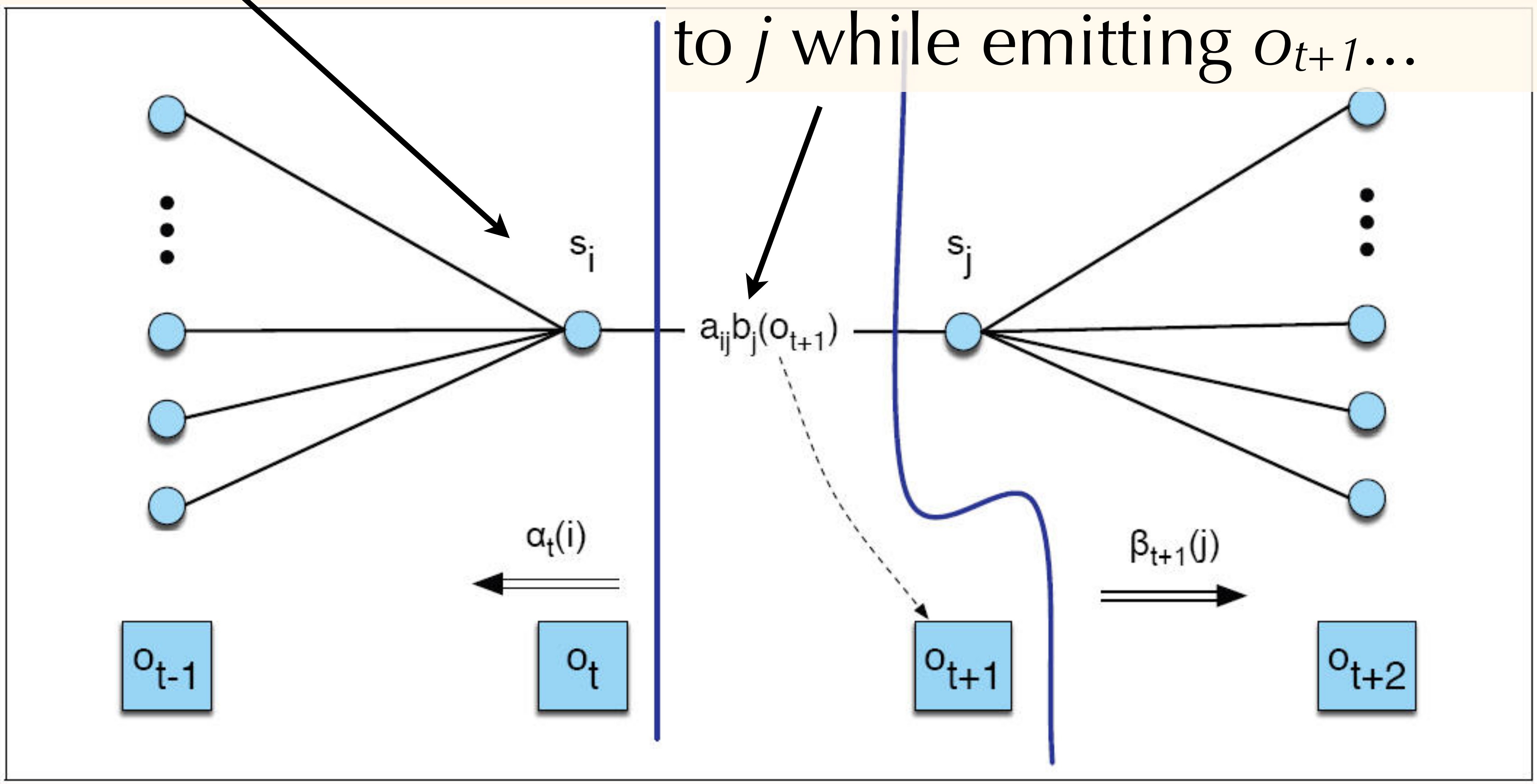

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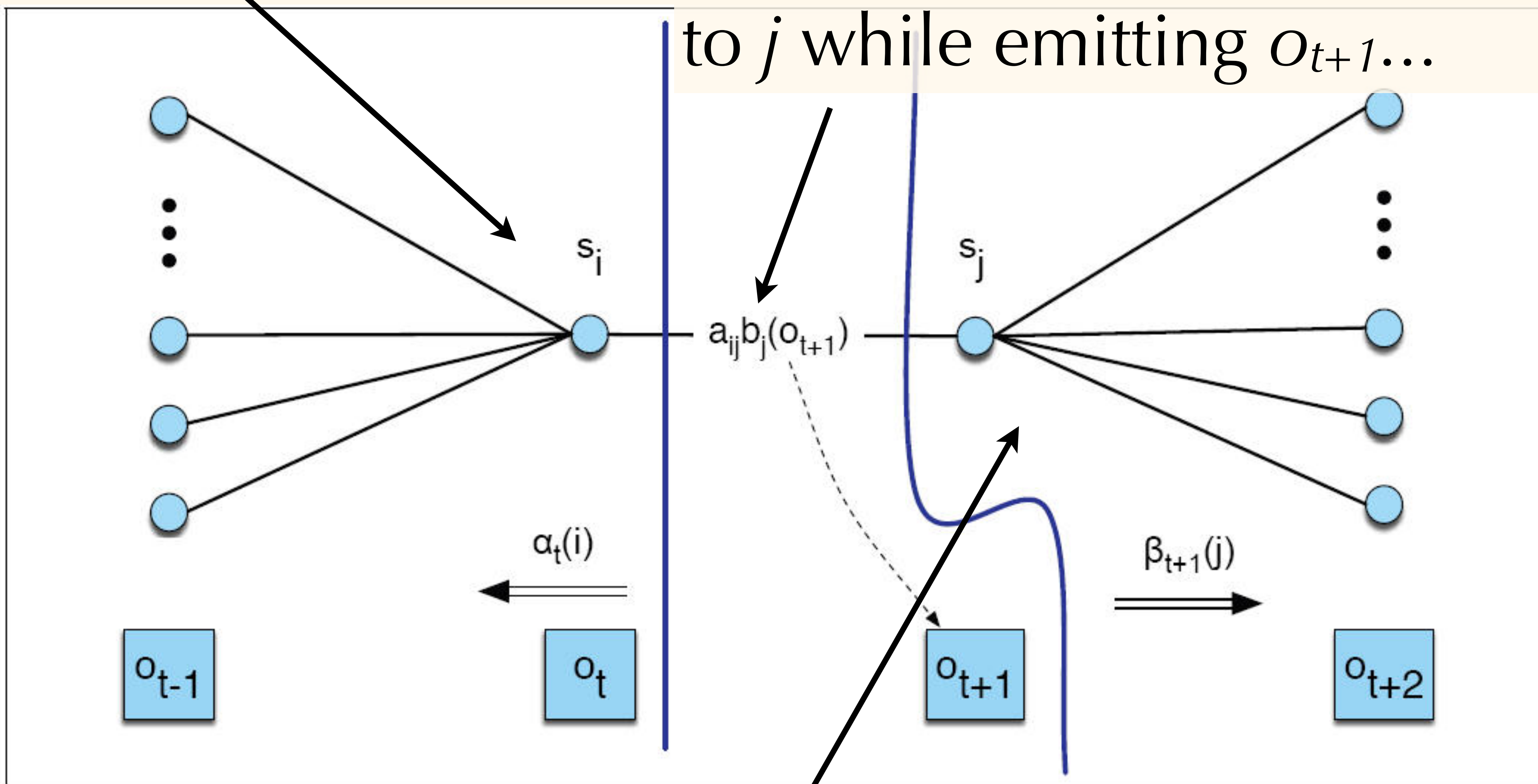

... all of the ways the model could finish from state *j* at time *t+1*.

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"Only count observations where the observed emission was v_k."

Now, that we have new estimates for *A* and *B*...

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{j=1}^{N} \xi_t(i,j)} \quad \hat{b}_j(v_k) = \frac{\sum_{t=1 \mathrm{s.t.} O_t = v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$$

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In practice, much depends on our initial estimates, and so we often use additional information when possible (e.g., encoding impossible transitions, etc.).