Vector Quantization, Gaussian Mixtures, and EM

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What is VQ?

- Representation of data in terms of codewords
- A data point is represented as the index of the nearest codeword
- In 2-D you end up storing one number, not two

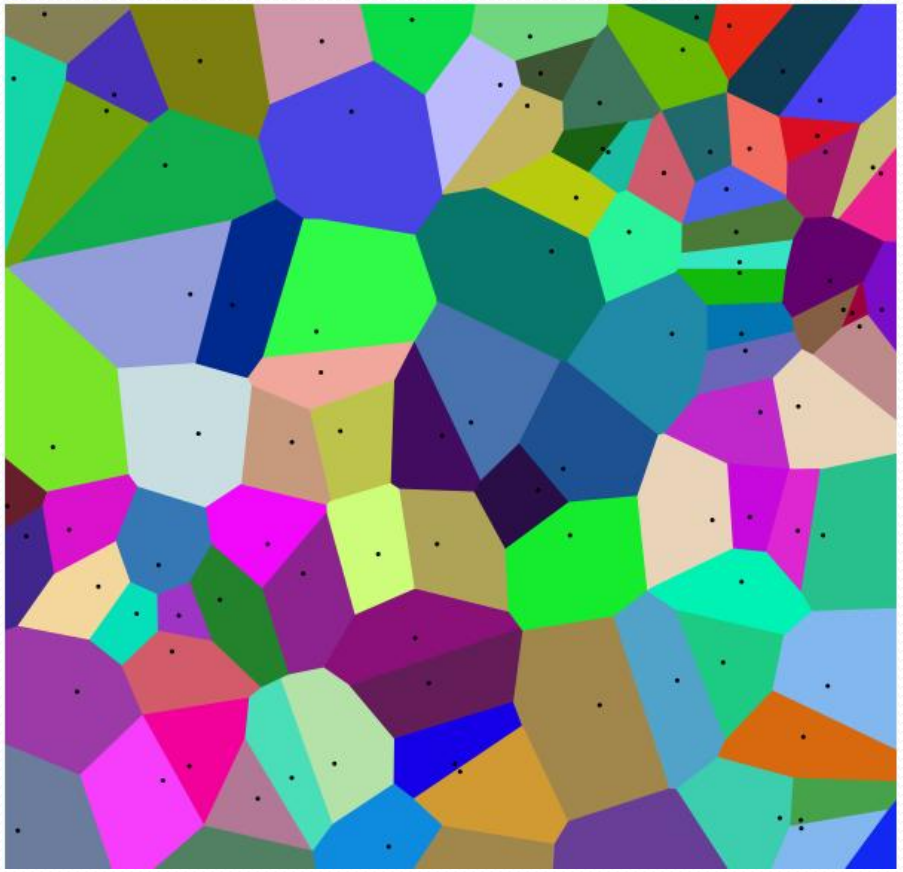


Image from Wikipedia

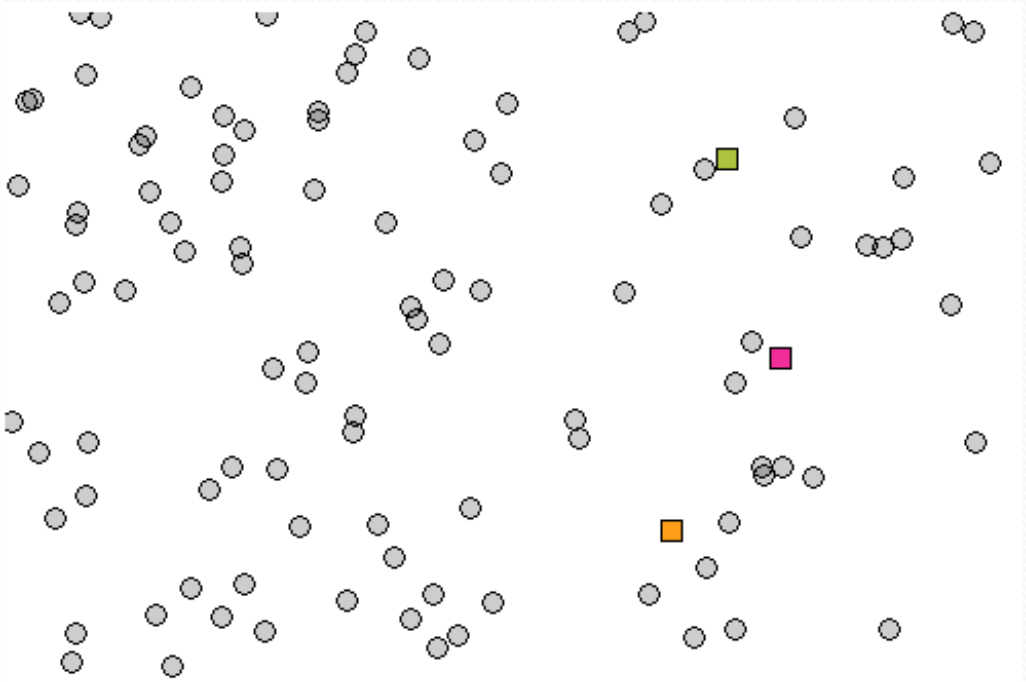
Why is it Useful?

- Data compression
- Data transmission
- Discrete representation of data is convenient to work with:
  - Enumerated probability distributions over single events
  - Language models and discrete HMMs for sequences

K-Means Algorithm

- Common form of vector quantization
- Creates K centers
- Initialization:
  - Choose K distinct points at random for the first centers
- Repeat:
  - Assign each data point to the nearest center
  - Reset each center to the mean of the points assigned to it
- Stopping criteria can be:
  - an absolute number of iterations
  - or threshold on sum of all distances between centers and assigned points (total distortion)

Example of K-Means Algorithm



- http://home.dei.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html

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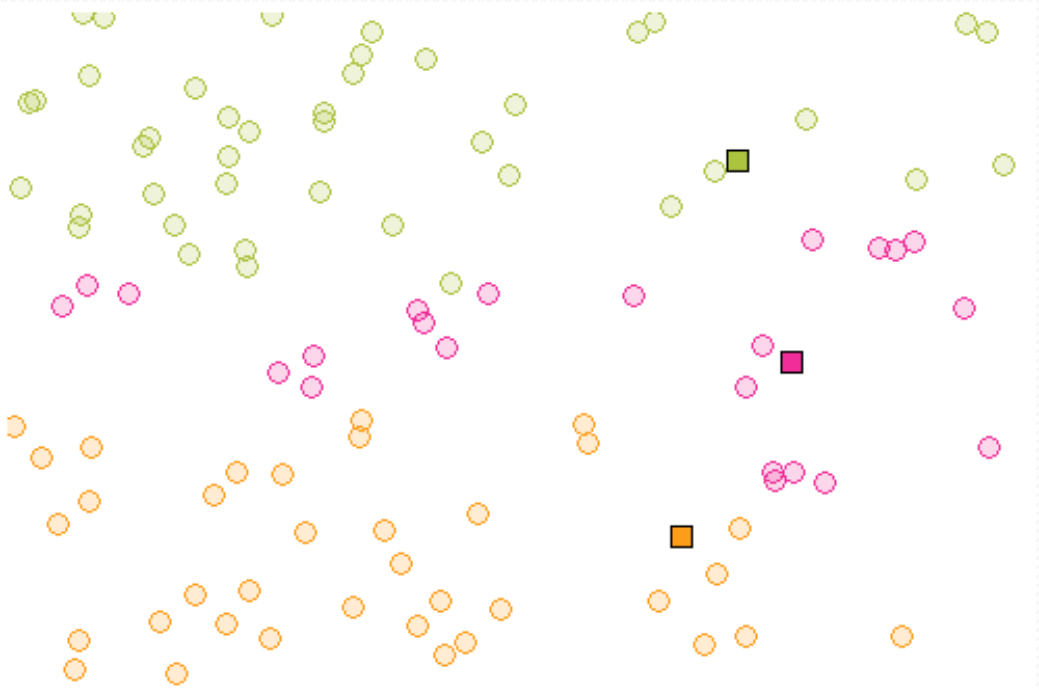

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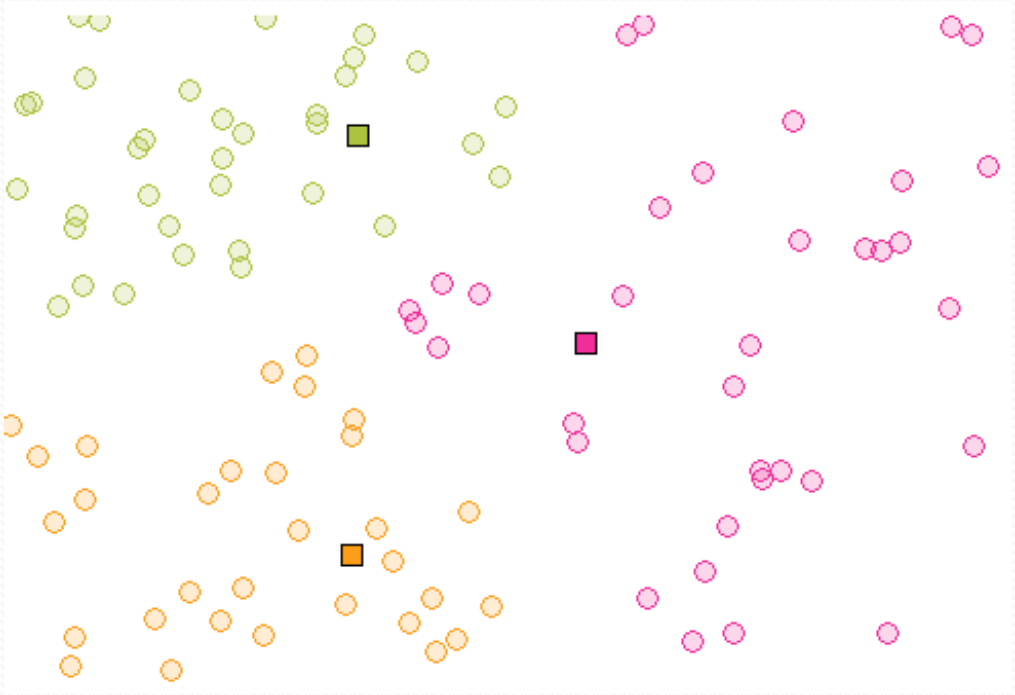

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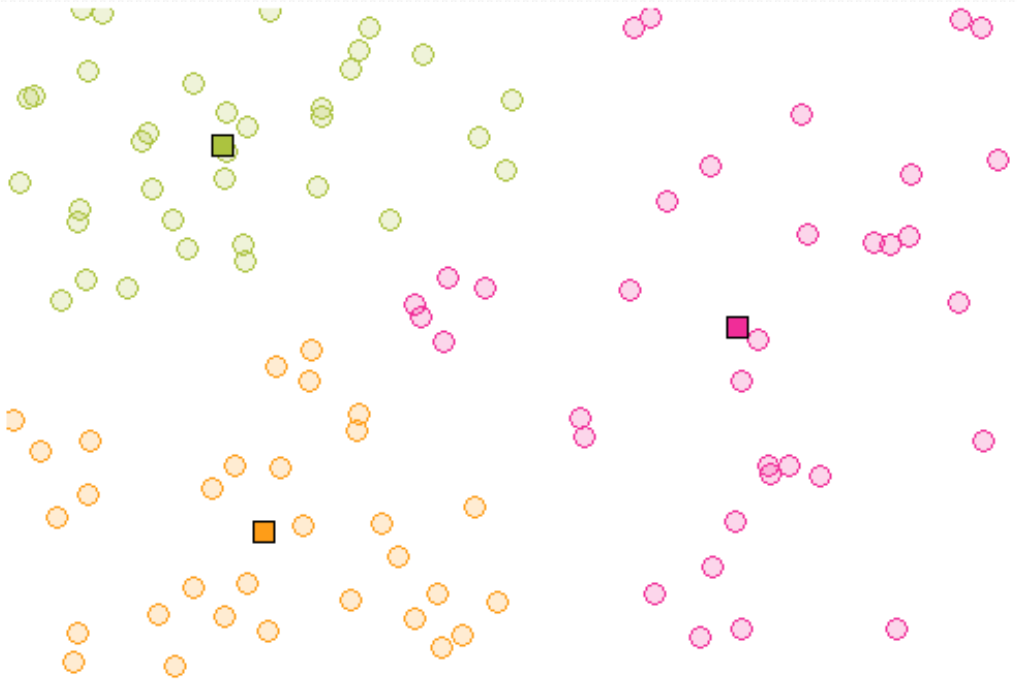

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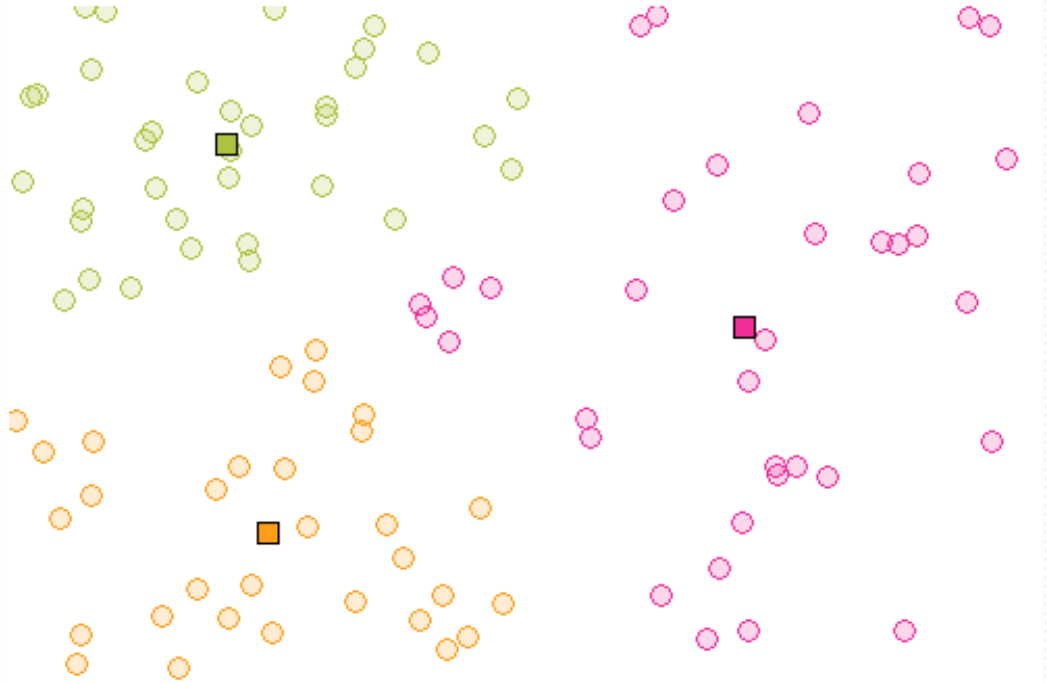

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VQ Convergence

- Consider the total "distortion"
- Sum of distances between points and their assigned centers

$$\sum_i (x_i - c(x_i))^2$$

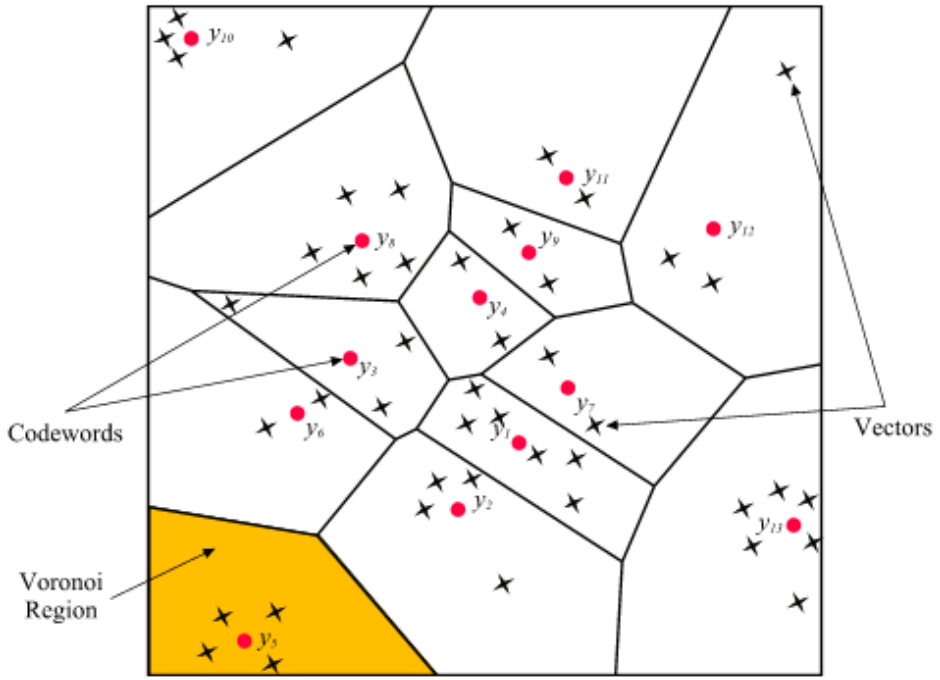


Image from http://www.geocities.com/mohamedqasem/vectorquantization/vq.html

Step 1: Assign each point to the nearest center

Defines C(xi) explicitly to minimize $(x_i - c(x_i))^2$

Since the contribution of each point individually to the distortion goes down, the total distortion must decrease

Step 2: Re-estimate each center as the mean of its assigned points

- Consider what happens to one center *c*

$$\frac{d}{dc}\sum_{i=1}^{N}(x_i - c)^2 = 0$$

$$\sum_{i=1}^{N}\frac{d}{dc}(x_i - c)^2 = 0$$

$$\sum_{i=1}^{N}2(x_i - c) = 0$$

$$c = \frac{1}{N}\sum_{i=1}^{N}x_i$$

Distortion contributed by each c individually is minimized

=> The total distortion is minimized

LBG

- Iteratively increases the number of codewords – 2,4,8,16…
- Can be used to induce a tree structured quantizer

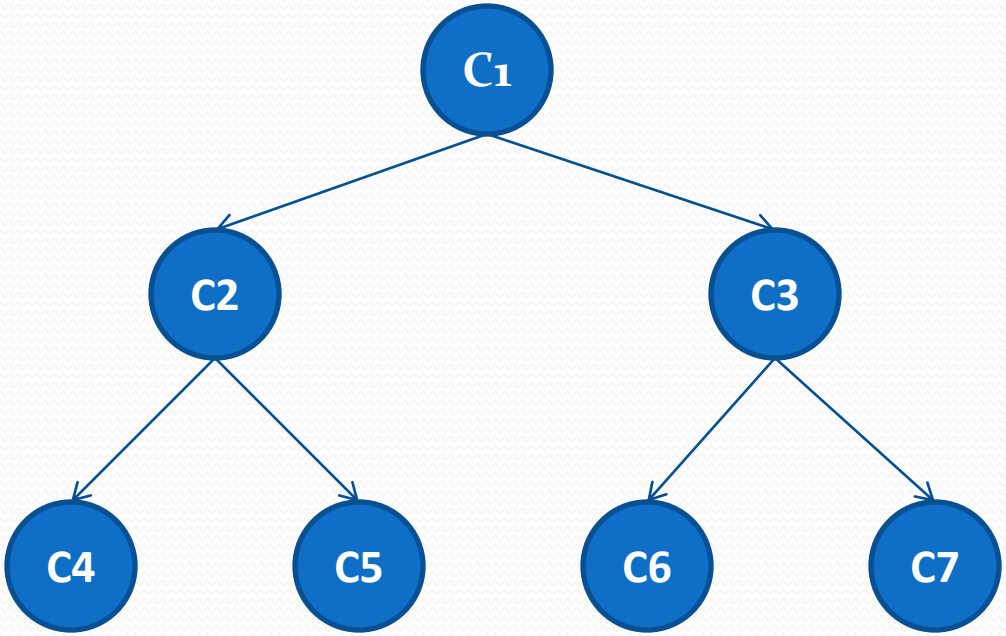
- Initialize:
  - Make one codeword in the center of everything
  - Assign all the data to it
- Repeat:
  - Split each current codeword into two slightly different variants
  - Do k-means with the current codewords

LBG Example

- http://www.data-compression.com/vq.shtml

VQ: Some Things to be Aware Of

Speeding Up VQ with a Tree



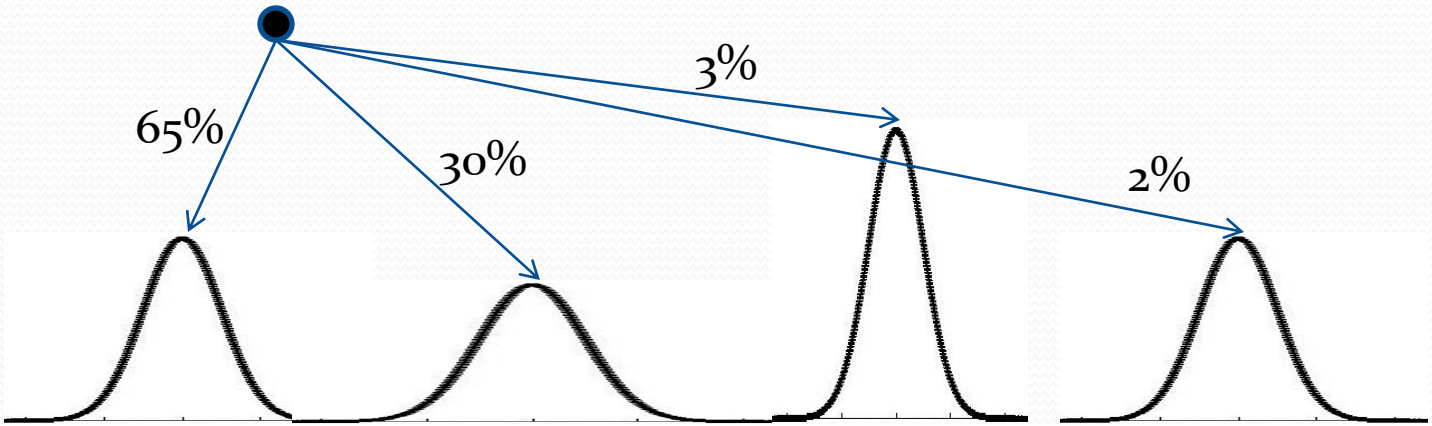
Recursively partition the data as the tree is built

Only follow one branch when finding a codeword after the tree is built

Gaussian Mixtures & EM

Gaussian Mixtures

- Codebook centers are gaussians
- An example may be assigned partially to a center
- A generative model
- Implies a data likelihood



Gaussian Refresher

$$N(x;\mu,\Sigma) = (2\pi)^{-d/2}|\Sigma|^{-1/2}\exp\left(-\frac{1}{2}(x-\mu)^t\Sigma^{-1}(x-\mu)\right)$$

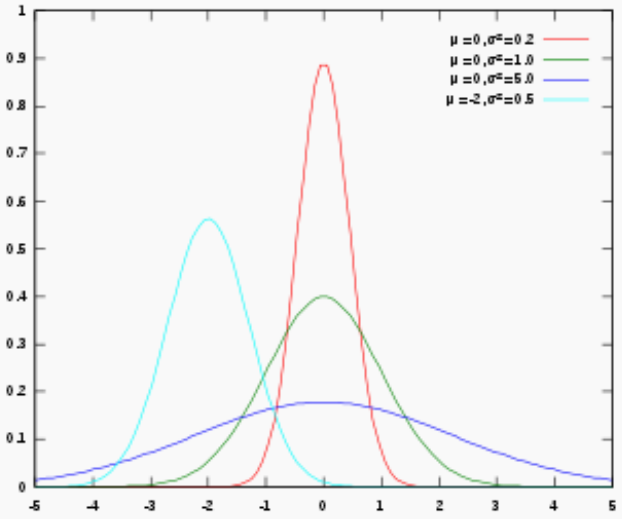


Image from Wikipedia

- Parameterized by mean and covariance matrix
- Integral over all space is 1 (probability density function)
- Diagonal covariance matrix most common in speech
  - O(d) parameters rather than O(d^2)

Maximum Likelihood Parameter Estimation (MLE) – Data Likelihood

$$\log p_n(\mathbf{x} \mid \mathbf{\Phi}) = \sum_{k=1}^{n} \log p(x_k \mid \mathbf{\Phi})$$

$$= \sum_{k=1}^{n} \log\left(\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x_k - \mu)^2}{2\sigma^2} \right] \right)$$

$$= -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{k=1}^{n}(x_k - \mu)^2$$

(n 1-dimensional points)

From Acero et al. Chapter 3

MLE – Take the Derivative

$$\frac{\partial}{\partial \mu} \log p_n(x \mid \mathbf{\Phi}) = \sum_{k=1}^{n} \frac{1}{\sigma^2}(x_k - \mu)$$

$$\frac{\partial}{\partial \sigma^2} \log p_n(x \mid \mathbf{\Phi}) = -\frac{n}{2\sigma^2} + \sum_{k=1}^{n} \frac{(x_k - \mu)^2}{2\sigma^4}$$

Set it equal to 0 and solve

$$\mu_{MLE} = \frac{1}{n} \sum_{k=1}^{n} x_k = E(x)$$

$$\sigma^2_{MLE} = \frac{1}{n} \sum_{k=1}^{n} (x_k - \mu_{MLE})^2 = E\left[(x - \mu_{MLE})^2 \right]$$

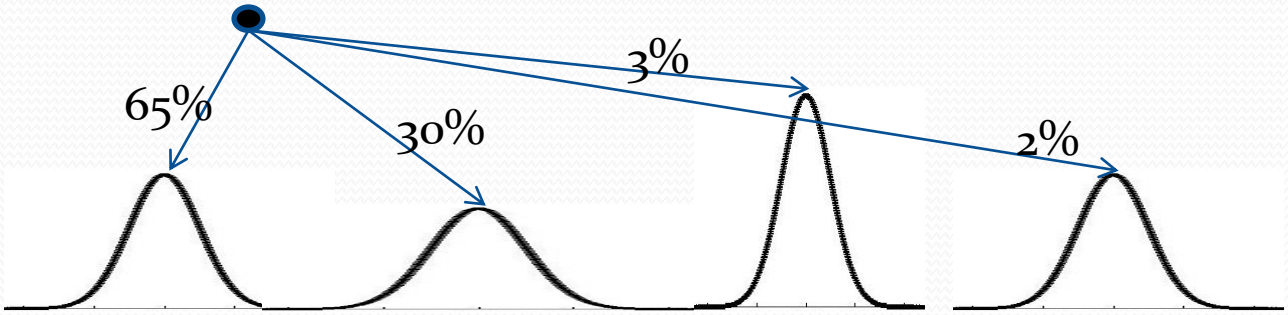
From Acero et al. Chapter 3

Why Do We Care About Gaussians

- A single gaussian is highly restricted
- But with enough gaussians you can model any probability distribution
- => A parametric modeling approach that becomes non-parametric
- And they are well understood in terms of
  - Parameter estimation
  - Computational complexity (and speedups)
  - Discriminative training
  - Adaptation to new data sets

K-Means for GMMs

- Same process as for VQ, but “soft” assignment
- Repeat:
  - Assign each data point to each gaussian with some weight
  - Re-estimate the gaussian centers using the weighted data assigned to each



MLE with GMMs

- Where we are going:
  - Parameter estimation will be as before
  - But the x_k s below will be weighted by "degree of membership"
  - And n will be the sum of the weights

$$\mu_{MLE} = \frac{1}{n}\sum_{k=1}^{n} x_k = E(x)$$

$$\sigma^2_{MLE} = \frac{1}{n}\sum_{k=1}^{n} (x_k - \mu_{MLE})^2 = E\left[(x - \mu_{MLE})^2\right]$$

From Acero et al. Chapter 3

Convergence of EM Process

Analysis will follow Sean Borman, "The Expectation Maximization Algorithm A Short Tutorial"
See also:
* Jeff Bilmes "A Gentle Tutorial on the EM Algorithm" and
* Acero et al. Chapter 4.

The data likelihood will go up at each iteration, analogous to distortion going down

$$L(\theta) = \ln \mathcal{P}(\mathbf{X}|\theta)$$

$$\mathcal{P}(\mathbf{X}|\theta) = \sum_{\mathbf{z}} \mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)$$

Hidden variables – what gaussian a data point comes from

Images from http://www.seanborman.com/publications/EM_algorithm.pdf

Jensen's Inequality

$$\ln \sum_{i=1}^{n} \lambda_i x_i \geq \sum_{i=1}^{n} \lambda_i \ln(x_i).$$

All λs must be non-negative and sum to 1

Change in Likelihood

$$
\begin{aligned}
L(\theta) - L(\theta_n) &= \ln\left(\sum_{\mathbf{z}} \mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)\right) - \ln \mathcal{P}(\mathbf{X}|\theta_n) \\
&= \ln\left(\sum_{\mathbf{z}} \mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta) \cdot \frac{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)}{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)}\right) - \ln \mathcal{P}(\mathbf{X}|\theta_n) \\
&= \ln\left(\sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)\frac{\mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)}{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)}\right) - \ln \mathcal{P}(\mathbf{X}|\theta_n) \\
&\geq \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln\left(\frac{\mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)}{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)}\right) - \ln \mathcal{P}(\mathbf{X}|\theta_n) \quad (12) \\
&= \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln\left(\frac{\mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)}{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)\mathcal{P}(\mathbf{X}|\theta_n)}\right) \quad (13) \\
&\triangleq \Delta(\theta|\theta_n). \quad (14)
\end{aligned}
$$

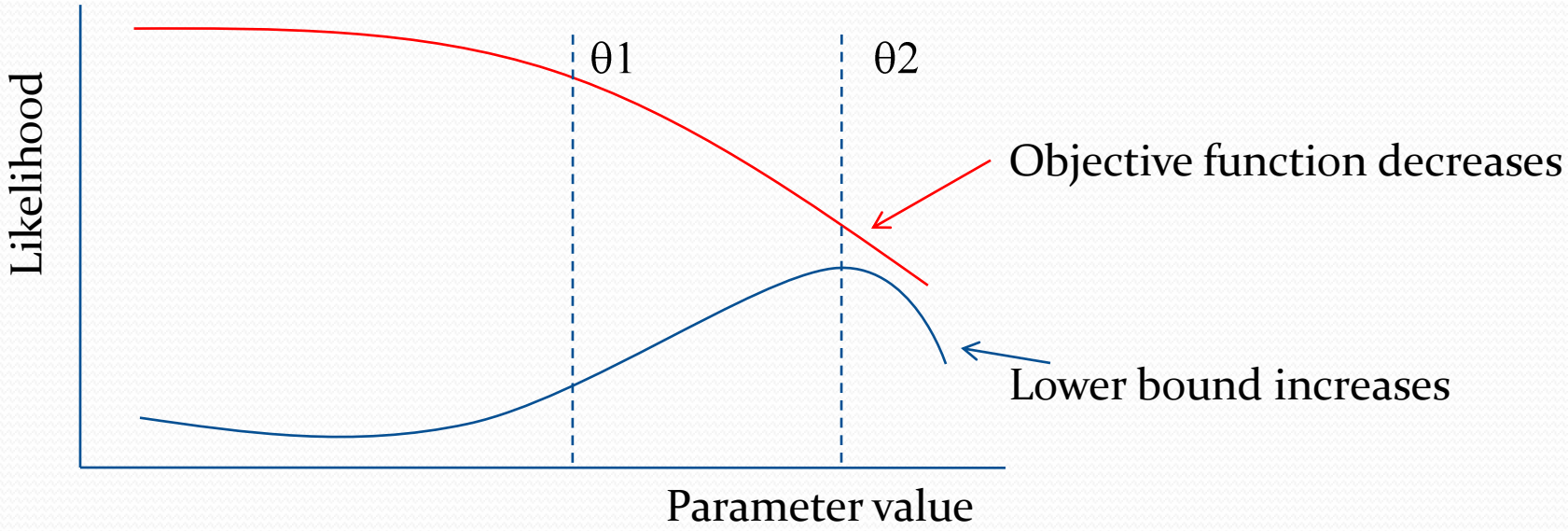
In going from Equation (12) to Equation (13) we made use of the fact that $\sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) = 1$ so that $\ln \mathcal{P}(\mathbf{X}|\theta_n) = \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \mathcal{P}(\mathbf{X}|\theta_n)$ which allows the term $\ln \mathcal{P}(\mathbf{X}|\theta_n)$ to be brought into the summation.

From http://www.seanborman.com/publications/EM_algorithm.pdf

Lower Bound on New Likelihood

$$L(\theta) \geq L(\theta_n) + \Delta(\theta|\theta_n) \triangleq l(\theta|\theta_n)$$

We'll work by increasing this lower bound.
But will increasing a lower bound increase what we want?

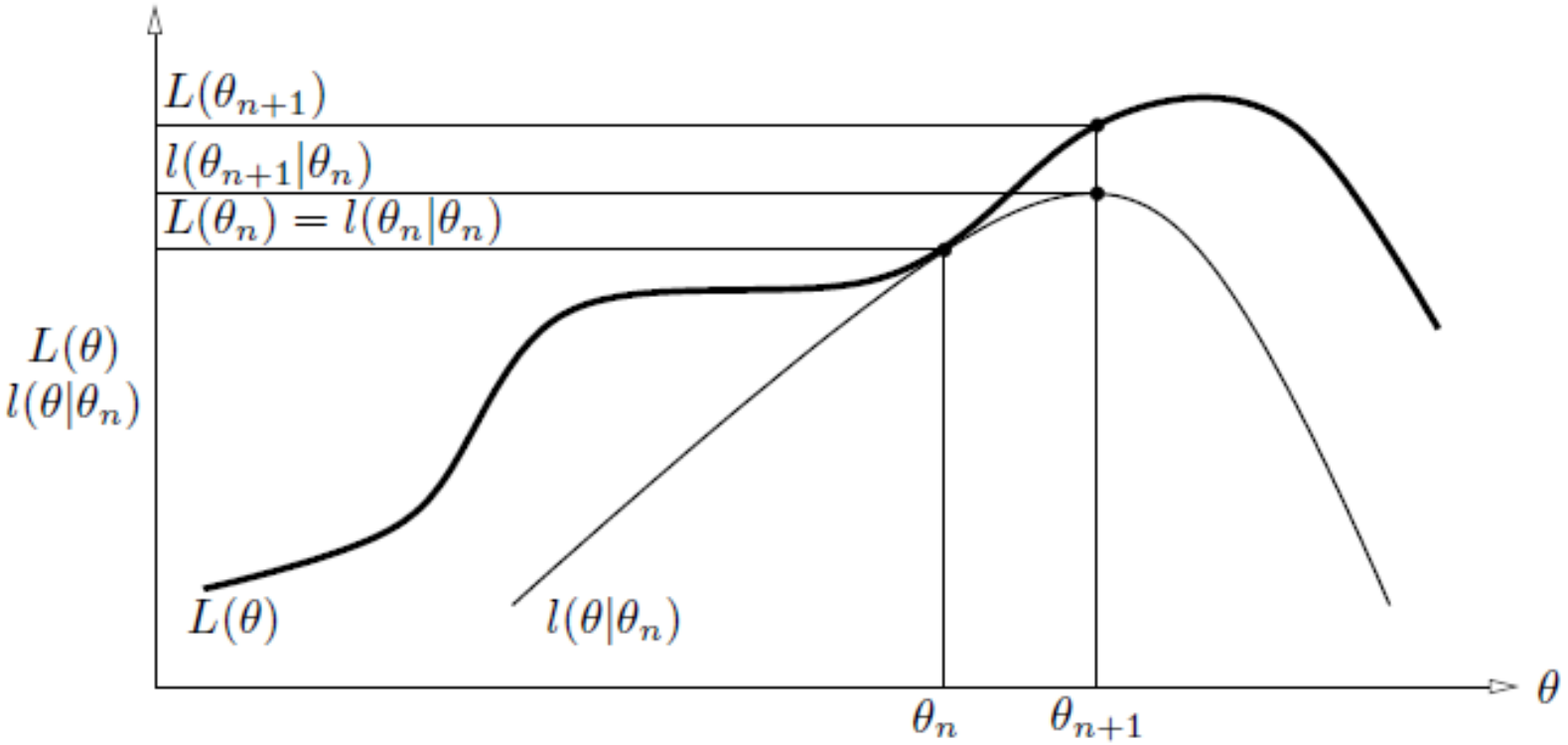


Lower Bound Evaluated at Current Parameters is the Likelihood Itself!

$$
\begin{aligned}
l(\theta_n|\theta_n) &= L(\theta_n) + \Delta(\theta_n|\theta_n) \\
&= L(\theta_n) + \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \frac{\mathcal{P}(\mathbf{X}|\mathbf{z},\theta_n)\mathcal{P}(\mathbf{z}|\theta_n)}{\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)\mathcal{P}(\mathbf{X}|\theta_n)} \\
&= L(\theta_n) + \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \frac{\mathcal{P}(\mathbf{X},\mathbf{z}|\theta_n)}{\mathcal{P}(\mathbf{X},\mathbf{z}|\theta_n)} \\
&= L(\theta_n) + \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln 1 \\
&= L(\theta_n),
\end{aligned}
$$

From http://www.seanborman.com/publications/EM_algorithm.pdf

The Real Picture



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Maximizing the Lower Bound on Likelihood

$$
\begin{aligned}
\theta_{n+1} &= \arg\max_{\theta} \{l(\theta|\theta_n)\} \\
&= \arg\max_{\theta} \left\{ L(\theta_n) + \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \frac{\mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta)}{\mathcal{P}(\mathbf{X}|\theta_n)\mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n)} \right\} \\
&\quad \text{Now drop terms which are constant w.r.t. } \theta \\
&= \arg\max_{\theta} \left\{ \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \mathcal{P}(\mathbf{X}|\mathbf{z},\theta)\mathcal{P}(\mathbf{z}|\theta) \right\} \\
&= \arg\max_{\theta} \left\{ \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \frac{\mathcal{P}(\mathbf{X},\mathbf{z},\theta)}{\mathcal{P}(\mathbf{z},\theta)} \frac{\mathcal{P}(\mathbf{z},\theta)}{\mathcal{P}(\theta)} \right\} \\
&= \arg\max_{\theta} \left\{ \sum_{\mathbf{z}} \mathcal{P}(\mathbf{z}|\mathbf{X},\theta_n) \ln \mathcal{P}(\mathbf{X},\mathbf{z}|\theta) \right\} \\
&= \arg\max_{\theta} \{ \mathrm{E}_{\mathbf{z}|\mathbf{X},\theta_n} \{\ln \mathcal{P}(\mathbf{X},\mathbf{z}|\theta)\} \}
\end{aligned}
$$

Prob. of z wrt current parameters

The Q Function

Full data likelihood wrt new params

From http://www.seanborman.com/publications/EM_algorithm.pdf

What This Tells Us

- Compute the expected values of the hidden variables
- Assume the hidden variables are seen with these probabilities
- Compute a new set of parameters θ to optimize the complete data likelihood
  - The Q function is a function of θ
  - It is maximized wrt θ
- This is guaranteed to increase the likelihood

Application to GMMs

$$p(\mathbf{y} \mid \mathbf{\Phi}) = \sum_{k=1}^{K} c_k p_k(\mathbf{y} \mid \mathbf{\Phi}_k) = \sum_{k=1}^{K} c_k N_k(\mathbf{y} \mid \boldsymbol{\mu}_k, \mathbf{\Sigma}_k)$$

One data point, K gaussians

$$\gamma_k^i = \frac{c_k p_k(\mathbf{y}_i \mid \mathbf{\Phi}_k)}{P(\mathbf{y}_i \mid \mathbf{\Phi})}$$

Posterior probability (count) of gaussian k wrt data point i

$$\gamma_k = \sum_{i=1}^{N} \gamma_k^i = \sum_{i=1}^{N} \frac{c_k p_k(\mathbf{y}_i \mid \mathbf{\Phi}_k)}{P(\mathbf{y}_i \mid \mathbf{\Phi})}$$

Total number of points assigned to Gaussian k

From Acero et al. Chapter 4

Application to GMMs (2)

$$\hat{c}_k = \frac{\gamma_k}{\sum_{k=1}^{K} \gamma_k} = \frac{\gamma_k}{N}$$

New prior for gaussian k

$$\hat{\boldsymbol{\mu}}_k = \frac{\sum_{i=1}^{N} \gamma_k^i \mathbf{y}_i}{\sum_{i=1}^{N} \gamma_k^i} = \frac{\sum_{i=1}^{N} \frac{c_k p_k(\mathbf{y}_i \mid \boldsymbol{\Phi}_k)\mathbf{y}_i}{P(\mathbf{y}_i \mid \boldsymbol{\Phi})}}{\sum_{i=1}^{N} \frac{c_k p_k(\mathbf{y}_i \mid \boldsymbol{\Phi}_k)}{P(\mathbf{y}_i \mid \boldsymbol{\Phi})}}$$

Mean is posterior-weighted average of the points

$$\hat{\boldsymbol{\Sigma}}_k = \frac{\sum_{i=1}^{N} \gamma_k^i (\mathbf{y}_i - \boldsymbol{\mu}_k)(\mathbf{y}_i - \boldsymbol{\mu}_k)^t}{\sum_{i=1}^{N} \gamma_k^i} = \frac{\sum_{i=1}^{N} \frac{c_k p_k(\mathbf{y}_i \mid \boldsymbol{\Phi}_k)(\mathbf{y}_i - \boldsymbol{\mu}_k)(\mathbf{y}_i - \boldsymbol{\mu}_k)^t}{P(\mathbf{y}_i \mid \boldsymbol{\Phi})}}{\sum_{i=1}^{N} \frac{c_k p_k(\mathbf{y}_i \mid \boldsymbol{\Phi}_k)}{P(\mathbf{y}_i \mid \boldsymbol{\Phi})}}$$

Variance also a weighted sum

From Acero et al. Chapter 4

Break

- Then:
- Advanced topics in GMMs

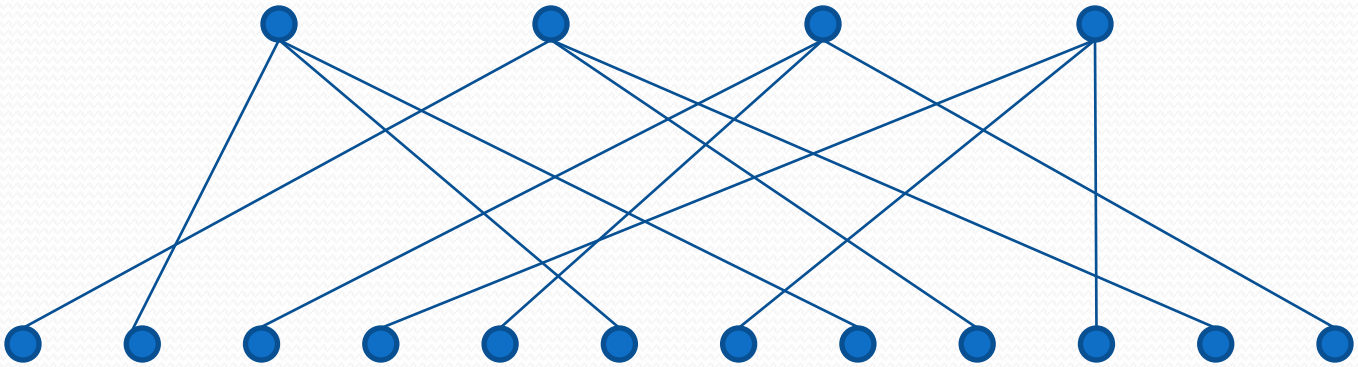
Fast Gaussian Computation

- Competition-grade systems may have close to 1M gaussians
- Typically features are extracted 100 times a second
- Evaluating and accumulating each dimension takes something like 2 additions and 2 multiplies
- 39 dimensions
- 100 million gaussian evaluations per second amounts to something like 15 billion ops/sec
- This is a problem for real-time or near real-time systems!

Some Options for Speeding Things Up

- On-Demand Computation
  - Only evaluate gaussians required by the search strategy
  - But: introduces linkage between search and gaussian computation, requires caching, and is complex
- Dimension-wise pruning
  - Likelihood computations involves sum of $((x-u)/\sigma)^2$ across dimensions – big number means low likelihood
  - Stop when you know the likelihood will be bad
  - But: limited benefit in practice
- Hierarchical evaluation
- Cache optimization

Hierarchical Evaluation



The gaussians we need to evaluate

Cluster them into a few high-level gaussians (e.g. 2000)

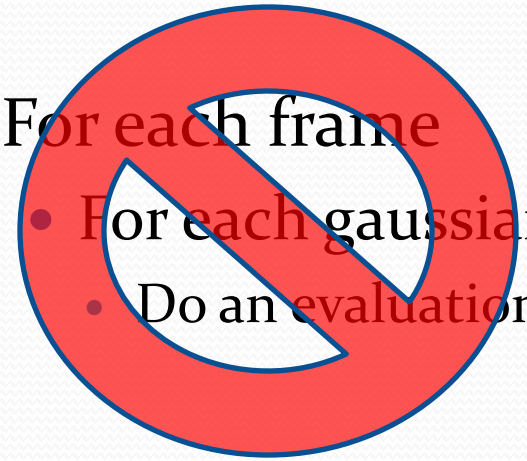
1. Evaluate the top level gaussians against a frame
2. Select the top N (e.g. 100)
3. Evaluate the "real" gaussians assigned to these top N
4. Assume everything else is zero
5. 20x speedup!

How to Cluster the Gaussians?

- K-Means of course!
- Some distance metrics:
  1. Euclidian distance between means
  2. KL-Divergence between a gaussian and the centroid

Cache Optimization

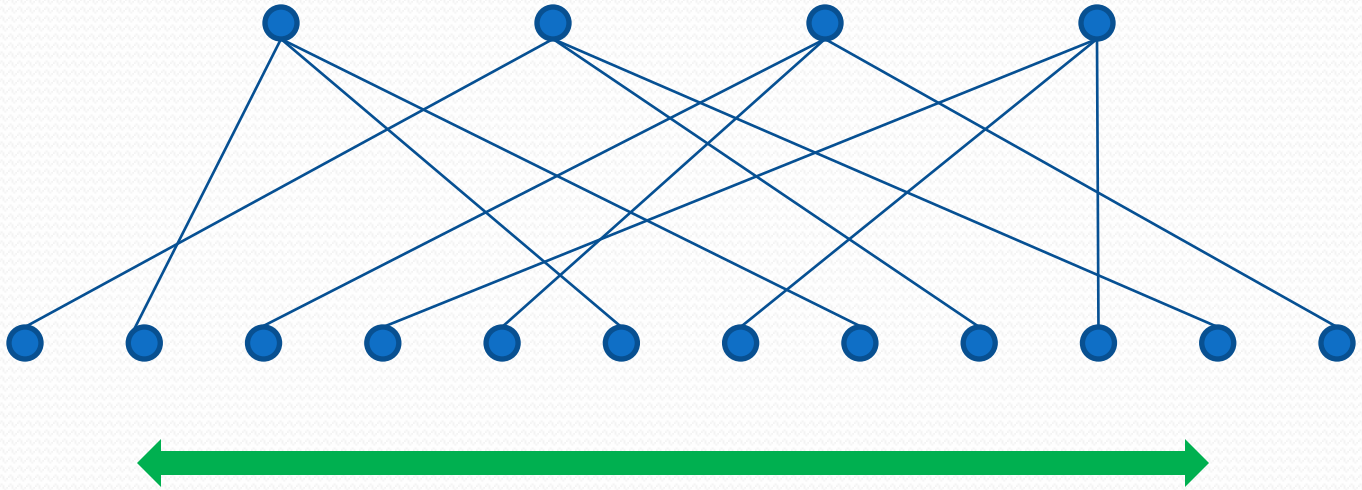
- For each frame
  - For each gaussian
    - Do an evaluation



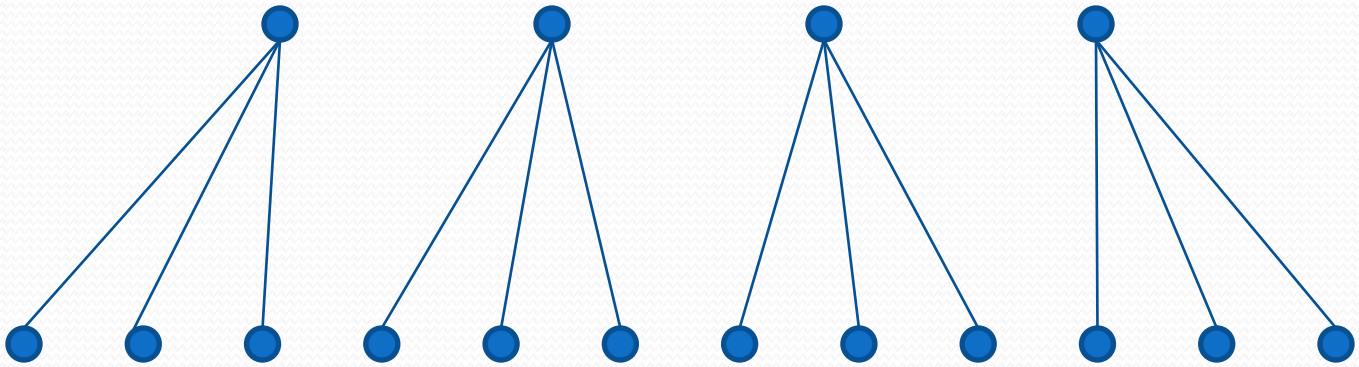
- For each gaussian
  - For each frame
    - Do an evaluation

Gaussians have means *and* variances
A frame takes ½ the memory!
½ as many cache misses
Maybe twice the speed
Applicable to hierarchical evaluation too

Cache Optimization (2)



Re-order for locality



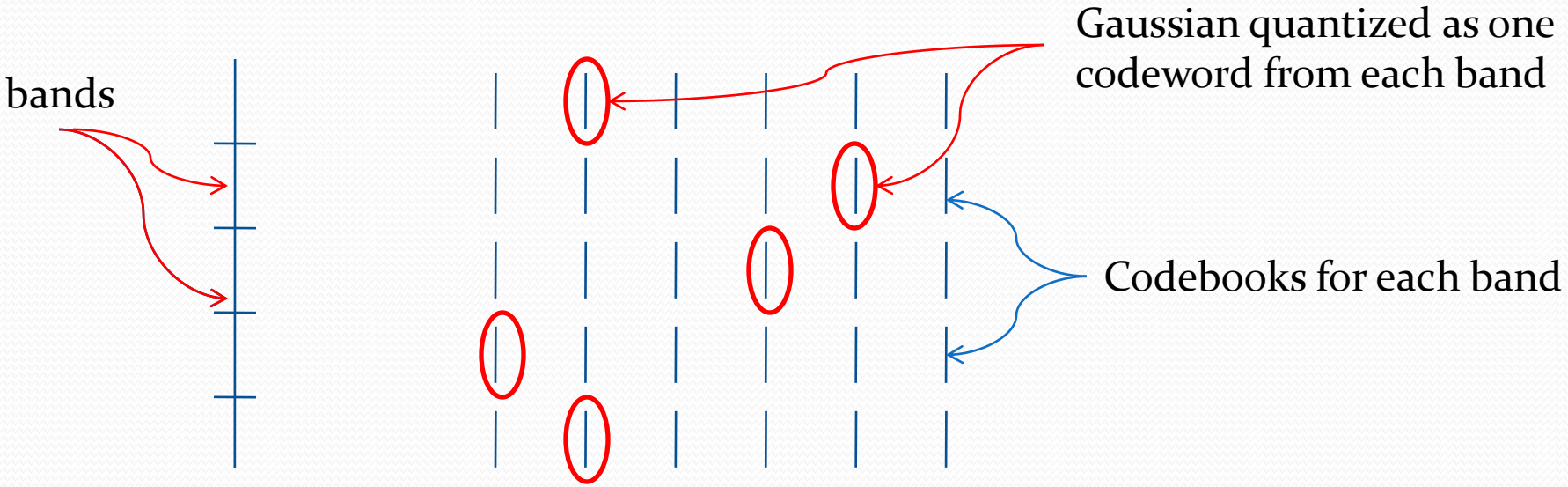
Low Memory Gaussian Computation

- Think circa 1990
- Dragon Dictate and IBM ViaVoice just introduced
- Think Intel 486
- 20MHz, 16MB RAM
- Memory was an issue!

- What to do?

Low Memory Gaussians (2)

- Break gaussians into bands
  - Each e.g. 2 dimensions
- Cluster all the samples in each band
  - Analogous to clustering the gaussians in the first place
- Diagonal covariance gaussians decompose into sum of bands
- Represent a gaussian as the sum of its bands



Consider 1-Dimensional Quantization

The quantized mean/variance of the d-th dimension of the j-th gaussian is:

$$\mu_d^{q(j)}, \sigma_d^{q(j)}$$

$$\log N^j(x;\mu,\Sigma) \propto D\log 2\pi + \sum_d \log\sigma_d^j + \sum_d (x_d - \mu_d^j)^2 (\sigma_d^j)^{-2}$$

$$\approx D\log 2\pi + \sum_d \log\sigma_d^{q(j)} + \sum_d (x_d - \mu_d^{q(j)})^2 (\sigma_d^{q(j)})^{-2}$$

Compute: $\log\sigma_d^{q(j)} + (x_d - \mu_d^{q(j)})^2 (\sigma_d^{q(j)})^{-2}$

Once for each codeword and re-use across gaussians

Memory Requirements

- Say 40 dimensions and bands are 2-dimensions
- Quantize to 256 codewords per band
- Each gaussian is now represented 20 bytes
- Used to be 40*2*4 =320 (assuming floats)
- Factor of 16 reduction

Compute requirements

- Evaluate 256*20 = 5120 2-dimensional gaussians
- Add 20 numbers to get the score for a “real” gaussian
- Repeatedly access the 5120 atomic numbers
  - Good for cache!

Further Speedups – Two References

- Aiyer, Gales & Picheny, "Rapid Likelihood Computation of Subspace Clustered Gaussian Components" (2000)
  - Many gaussians use common sets of codewords
  - Redundant computation
  - Can be optimized with compiler technology for evaluating common subexpressions once only
- Saon, Zweig & Povey, "Anatomy of an Extremely Fast LVCSR Decoder" (2003)
  - Numerous tricks for efficient organization of complete recognizer

Full Covariance Matrices

$$N(x;\mu,\Sigma) = (2\pi)^{-n/2}|\Sigma|^{-1/2}\exp\left(-\frac{1}{2}(x-\mu)^t\Sigma^{-1}(x-\mu)\right)$$

- When Σ^{-1} is not diagonal
  - Number of parameters is $O(D^2)$ not $O(D)$
  - Need more data to estimate the parameters
  - Evaluation is much slower
  - Band quantization doesn't work
  - Adaptation methods are more complex
- Nevertheless, people sometimes see improvements
  - EMLLT is an interesting compromise

EMLLT

$$\Sigma_j^{-1} = \sum_{k=1}^{D} \lambda_k^j a_k a_k^T$$

- Inverse covariance matrix is sum of outer products of basis vectors
- Can also think of as sum of basis matrices
- Basis vectors shared across all gaussians
  - Potentially many fewer covariance parameters – just D per gaussian
  - Plus the pool of basis vectors

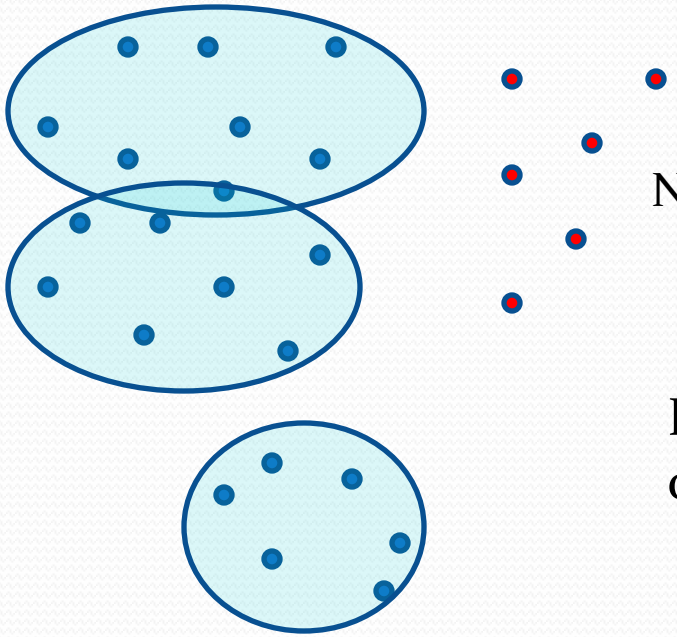
See Olsen & Gopinath, "Modeling Inverse Covariance Matrices by Basis Expansion"

Some EMLLT Results

Diagonal		MLLT		EMLLT		
n_{Gauss}	WER	n_{Gauss}	WER	n_{Gauss}	D	WER
10253	3.14%	10253	2.84%			
17028	3.08%	17028	2.74%	10253	$2d$	2.54%
26460	3.01%	26460	2.58%	10253	$4d$	2.34%
46500	2.84%	46500	2.50%	10253	$8d$	2.15%
				10253	$14d$	2.04%
				10253	$20d$	2.11%

d is the vector dimensionality
From Olsen & Gopinath,
"Modeling Inverse Covariance Matrices by Basis Expansion"

Adaptation

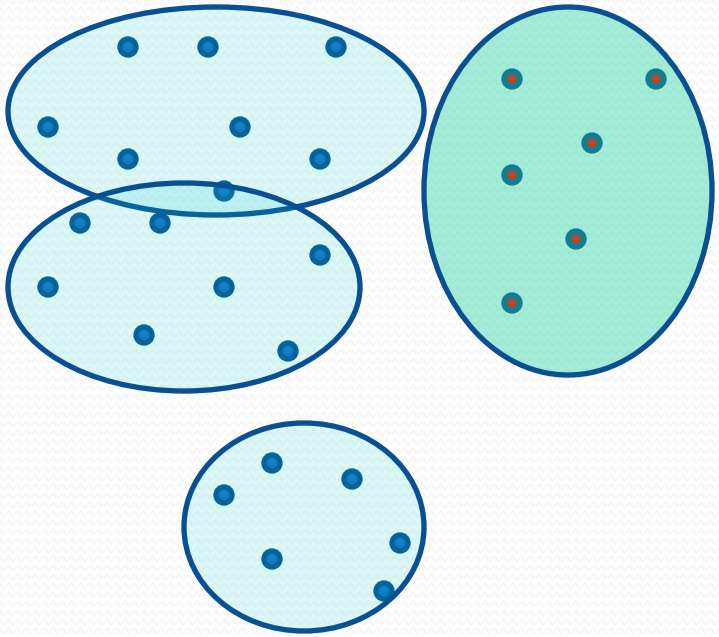


New data

How should we update our estimate of what the gaussians are?

Old data modeled by some gaussians

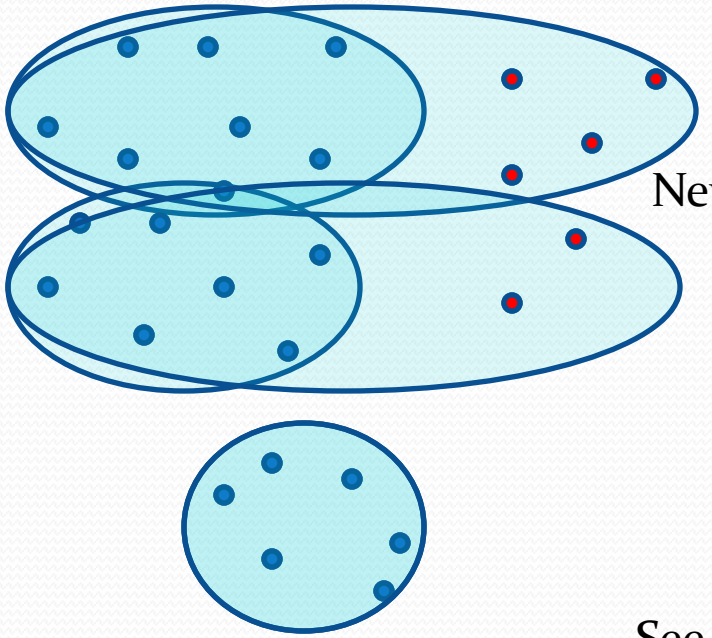
Option 1: Replace the old data



New data,
New gaussians

Old data modeled by
some gaussians

Option 2: Add the Data (MAP Adaptation)



New data

Old data modeled by some gaussians

Re-estimate gaussians, combining old and new data, possibly with a weighting factor

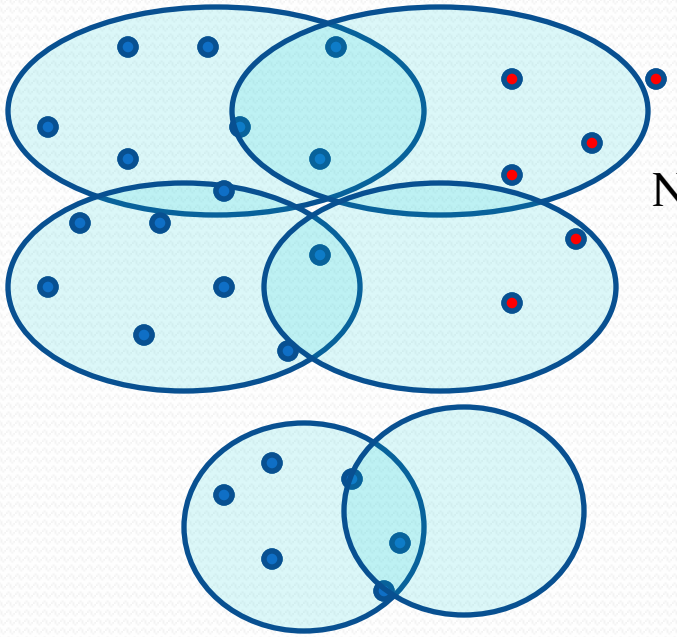
See, e.g. Gauvain & Lee, Maximum a Posteriori Estimation for Multivariate Gaussian Mixture Observations of Markov Chains

Option 3: Transform the Means

$$\mu' = A(1\ \mu^T)^T$$

- New mean is linear transformation of old
- An offset is added to the old mean as well
- Transformation matrix chosen to maximize the likelihood of the adaptation data under the transformed model
- One transformation (e.g. 39x39) shared by many gaussians (e.g. 1000s)
- See, e.g., Leggetter & Woodland, "Maximum likelihood linear regression for speaker adaptation of continuous density hidden Markov models"
- Similar transforms possible for covariance matrix

MLLR Picture

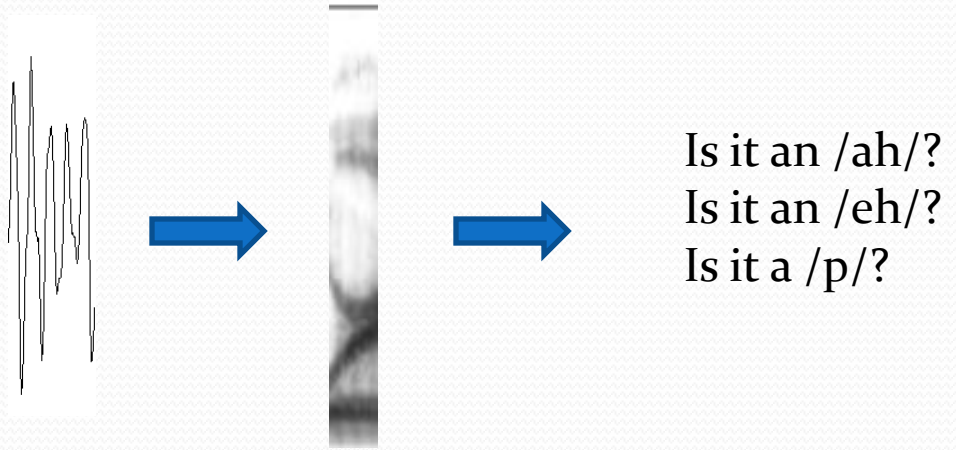


New data

New means are a linear transform of the old ones

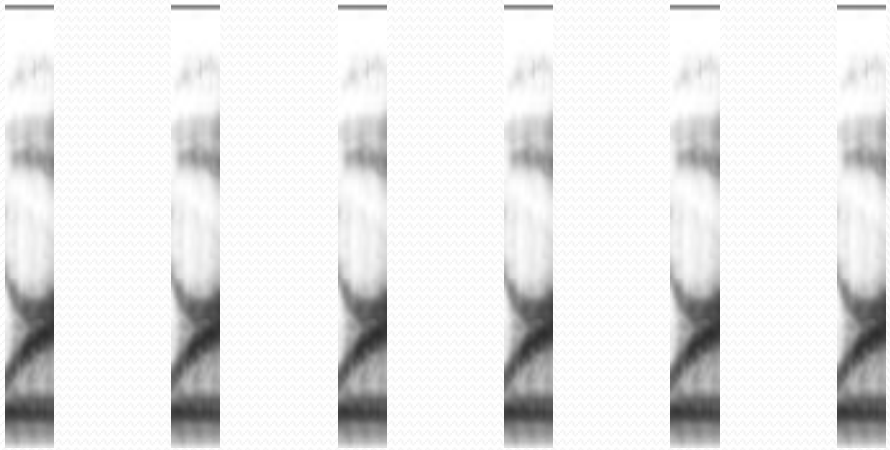
Old data modeled by some gaussians

Tying it All Together: Phone Probabilities



- Want: $\arg\max_q P(q \mid y) = \arg\max_q P(q)P(y \mid q)$
- Need to model P(y| phone q)
- Discrete (VQ) probabilities
- Continuous (GMM) probabilities
- Semi-Continuous probabilities

The Discrete Approach



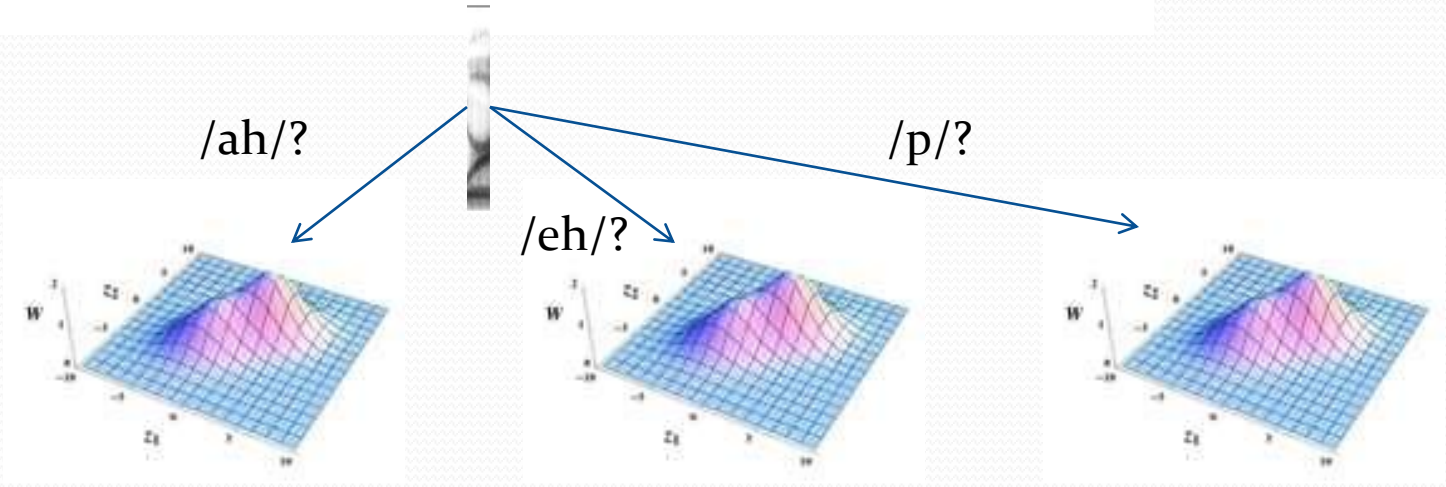
Note:
- Spectral slices should change
- MFCCs would normally be used

- Vector-quantize the feature vectors
  - Every 10ms or so
- $P(y_t \mid /ah/) = P(27 \mid /ah/)$
  - Learned by counting examples
  - Covered in HMM lecture

Continuous Probabilities

- Each phone has its own gaussian mixture

$$p(\mathbf{y} \mid \mathbf{\Phi}) = \sum_{k=1}^{K} c_k p_k(\mathbf{y} \mid \mathbf{\Phi}_k) = \sum_{k=1}^{K} c_k N_k(\mathbf{y} \mid \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

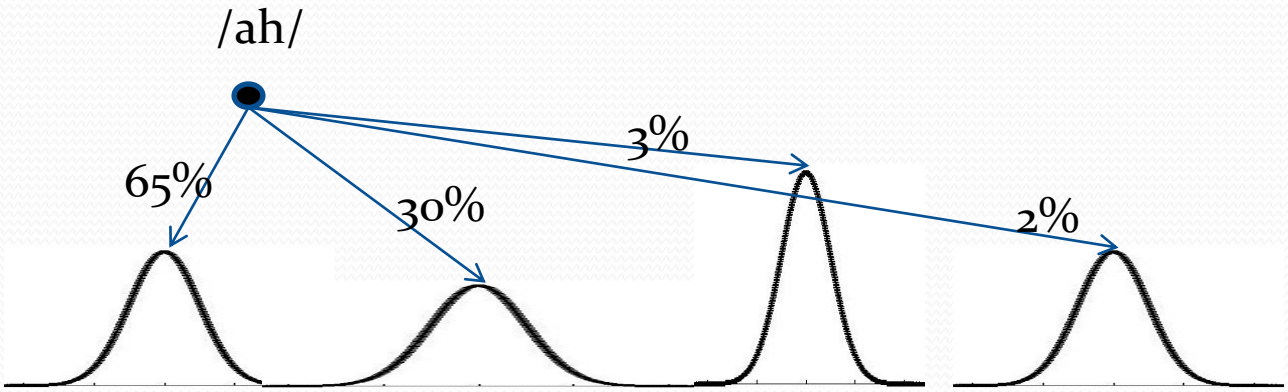


/ah/ gaussians /eh/ gaussians /p/ gaussians

Image from http://oregonstate.edu/~hohenlop/Gaussianmix.jpg

Semi-Continuous Probabilities

- All models share the same gaussians
- Models differ only in the weight assigned to each
- Continuous gaussian models are a special case
  - With lots of zeros as coefficients
- Not much used anymore in ASR



Homework

- Write a VQ program for 2-dimensional data
  - First use Euclidean distortion D(x,y) between points x,y (eq'n. 4.77 of Acero, et al.), squared Euclidean distance
- Use the provided "points" file as input
- Make a plot of the input
- Fit 1,2,4,8 and 16 centroids to the data
  - Plot the centers on top of the data
- Now use log distance
  - Log(1 + D(x,y))
- Can you find an analytical update, guaranteed to reduce distortion?
- Find an update that is guaranteed to reduce distortion at each iteration (analytical or not)
- Fit 1, 2, 4, 8 and 16 centroids to the data
  - Plot the centers on top of the data
  - Plot the Euclidean and Log-distance centroids together
- Finally, train a mixture of 1,2,4,8 and 16 gaussians with these points (using either K-means or LBG). Be sure to adjust the variances.
  - Plot the positions of the centers.
- Turn in all 4 plots for 4 centers
- Turn in a printout of your program
- Turn in a printout of the total distortion after each iteration as the program runs

Project Reminder

- Please think about your projects!
- For Speech Recognition / Langauge ID / Speaker ID, please contact me
  - After class
  - By email gzweig@microsoft.com
- On 5/7 I'd like to meet with everyone doing a relevant project.