

*Upon this gifted age, in its dark hour  
Rains from the sky, a meteoric shower  
Of facts... they lie, unquestioned, uncombined.  
Wisdom enough to leach us of our ill  
Is daily spun; but there exists no loom  
To weave it into a fabric.*

*“Huntsman, What Quarry?” . 1939, Edna St. Vincent Millay*

# John Tukey

*If we need a short suggestion of what exploratory data analysis is, I would suggest that*

- 1. It is an attitude, AND*
- 2. A flexibility, AND*
- 3. Some graph paper (or transparencies, or both).*

*No catalog of techniques can convey a willingness to look for what can be seen, whether or not anticipated. Yet this is at the heart of exploratory data analysis. The graph paper-and transparencies-are there, not as a technique, but rather as a recognition that the picture-examining eye is the best finder we have of the wholly unanticipated.*



John Tukey  
1915–2000

Tukey, 1980 . “We need both explanatory and confirmatory” *The American Statistician* 34(1) 23-25

Tukey also invented/discovered: many statistical tests, the word “bit”, (maybe) the word “software”, the Fast Fourier Transform, etc.

# Stem and Leaf Plots

Class A		Class B	
Stem	Leaves	Stem	Leaves
4	9	4	
5	5, 7	5	2, 7
6	6, 6, 8	6	2, 5, 8, 8
7	2, 8, 8, 8	7	2, 5
8	4, 5, 7, 8, 8	8	1, 4, 5, 7, 7
9	1, 5, 5	9	0, 1, 1, 5, 5, 5
10	0, 0	10	0

Male		Female
5, 2, 0	1	5, 8
5, 1	2	1, 6, 9, 9
5, 5, 5, 3, 1	3	
5, 2	4	1, 2, 6, 8
9, 8, 6, 1, 1	5	5
6, 5, 5, 0	6	0, 1
2, 1, 1, 0, 0	7	2

Stem and leaf plots can be useful for quickly looking at relatively small amounts of data.

Of course, if you turn them sideways, you've got a histogram...



# みなとみらい線標準時刻表

Train Departures of Minatomirai Line

みなとみらい 元町・中華街 方面  
for Minatomirai, Motomachi · Chūkagai

平日 Weekdays

5	10 17 29 39 47 54
6	1 7 13 16 22 26 30 32 36 41 49 48 51 55 58
7	1 4 7 10 13 17 20 22 25 28 31 34 37 40 44 46 50 53 57 59
8	2 5 8 11 14 17 21 24 27 31 35 38 41 44 48 51 54 58
9	2 6 10 14 17 21 24 28 31 35 38 41 44 47 51 55 59
10	3 7 11 16 20 27 30 36 39 41 45 50 54 56
11	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
12	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
13	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
14	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
15	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
16	0 4 9 11 15 19 24 26 30 32 36 39 41 45 47 50 54 57
17	1 5 9 13 16 20 24 28 32 36 40 44 46 50 54 57
18	1 4 8 12 16 19 23 27 31 34 38 42 45 49 53 58
19	1 5 9 13 16 20 24 28 31 35 38 42 45 50 54 58
20	0 5 8 12 16 20 24 28 31 34 38 42 46 49 54 57
21	1 5 8 12 16 20 24 28 31 34 38 42 46 49 54 57
22	2 7 11 15 19 23 27 31 37 42 47 51 56 59
23	3 9 14 18 22 26 30 34 40 45 48 56
0	4 5 12 16 21 30 34 48

土曜日・休日 Saturdays, Sundays and Holidays

5	10 17 29 39 47 54
6	1 7 14 19 22 30 37 42 45 50 54 58
7	1 6 10 14 18 23 27 30 35 40 45 50 54 57
8	0 4 9 12 15 20 24 27 32 36 39 42 45 50 54 57
9	0 5 9 12 15 20 24 26 30 34 39 41 45 49 54 56
10	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
11	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
12	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
13	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
14	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
15	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
16	0 4 9 11 15 19 24 26 30 34 39 41 45 49 54 56
17	0 4 9 11 15 18 21 24 27 30 34 39 43 45 49 53 58
18	0 4 8 13 16 20 23 27 30 35 39 43 46 48 51 55 59
19	2 6 10 13 18 21 24 27 31 36 39 44 50 54 57
20	1 6 10 15 20 24 28 31 36 39 44 50 53 58
21	1 6 10 15 20 24 28 32 36 40 45 50 54 58
22	2 6 11 16 23 26 30 37 44 50 56
23	0 6 14 22 28 32 40 49 55 59
0	6 16 28 38

**時刻表の見方**

種別  
00 特急  
00 通勤特急  
00 急行  
00 各駅停車

行先  
元町・中華街  
00

行先の特典  
00 元町・中華街

※ 女性専用車のご案内 ※

平日7:00 始発～10:00迄  
土曜・日曜・17:00～19:00迄  
特急、通勤特急、急行列車は女性専用車とはなりません  
※ 女性専用車は1両目、2両目が利用できます





# みなとみらい線標準時刻表

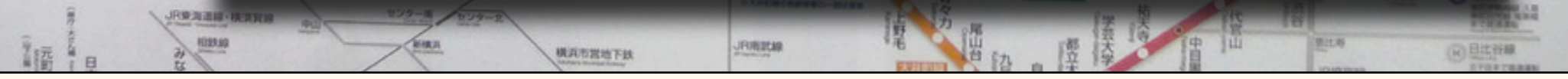
Train Departures of Minatomirai Line

みなとみらい 元町・中華街 方面  
for Minatomirai, Motomachi · Chūkagai 方面

平日 Weekdays

5	10 17 29 39 47 54
6	1 7 13 16 22 26 30
7	1 4 7 10 13 17 20 23
8	2 5 8 11 14 17 21 24
9	2 6 10 14 17 21 24 27
10	3 7 11 16 20 27 30 33
11	0 4 9 11 15 19 24 28
12	0 4 9 11 15 19 24 28
13	0 4 9 11 15 19 24 28
14	0 4 9 11 15 19 24 28
15	0 4 9 11 15 19 24 28
16	0 4 9 11 15 19 24 28
17	1 5 9 13 16 20 24 27
18	1 4 8 12 16 19 23 26
19	1 5 9 13 16 20 23 26
20	0 5 8 12 16 20 23 26
21	1 5 8 10 17 21 25 28
22	2 7 11 15 19 23 27 30
23	3 9 14 18 22 26 30 33
0	4 5 12 16 21 30 34

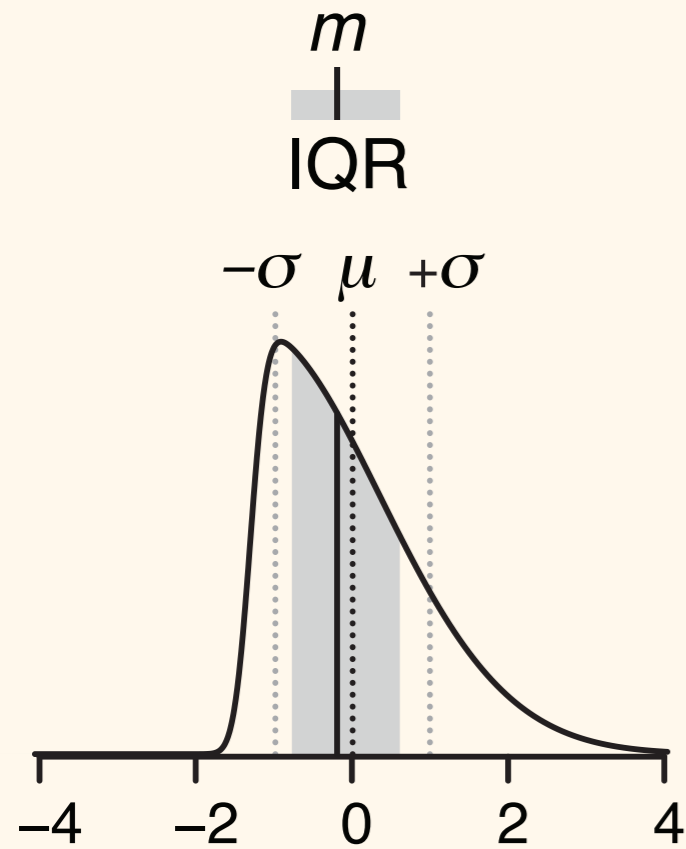
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18	0	4	8	13	16	20	23	27	30	35	39	43	46	48	51	55	59
19	2	6	10	13	18	21	24	27	31	36	39	44	50	54	57		
20	1	6	10	15	20	24	28	31	36	39	44	50	53	58			
21	1	6	10	15	20	24	28	32	36	40	45	50	54	58			
22	2	6	11	16	23	26	30	37	44	50	56						
23	0	6	14	22	28	32	40	49	55	59							
0	6	16	28	38													



# Boxplots (also invented by Tukey)

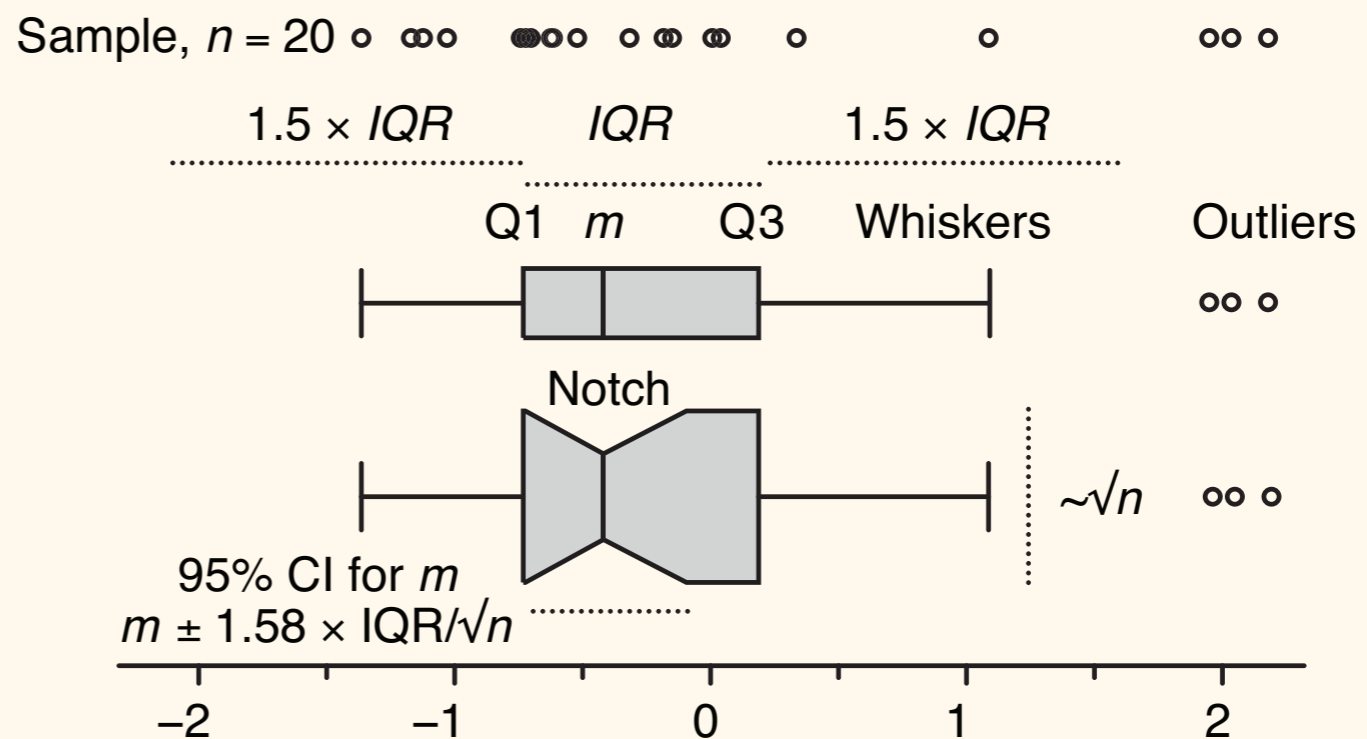
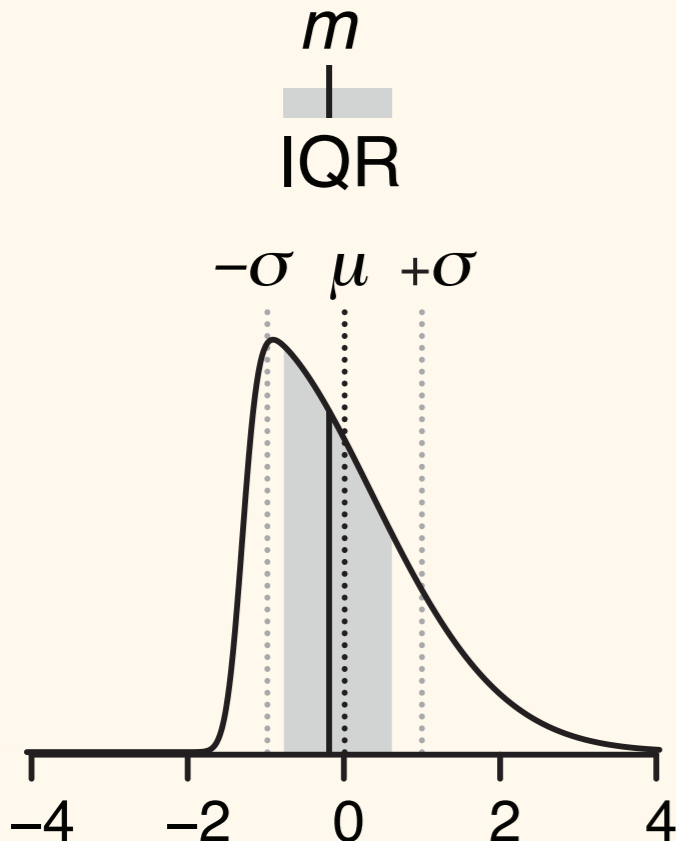
# Boxplots (also invented by Tukey)

Population distribution



# Boxplots (also invented by Tukey)

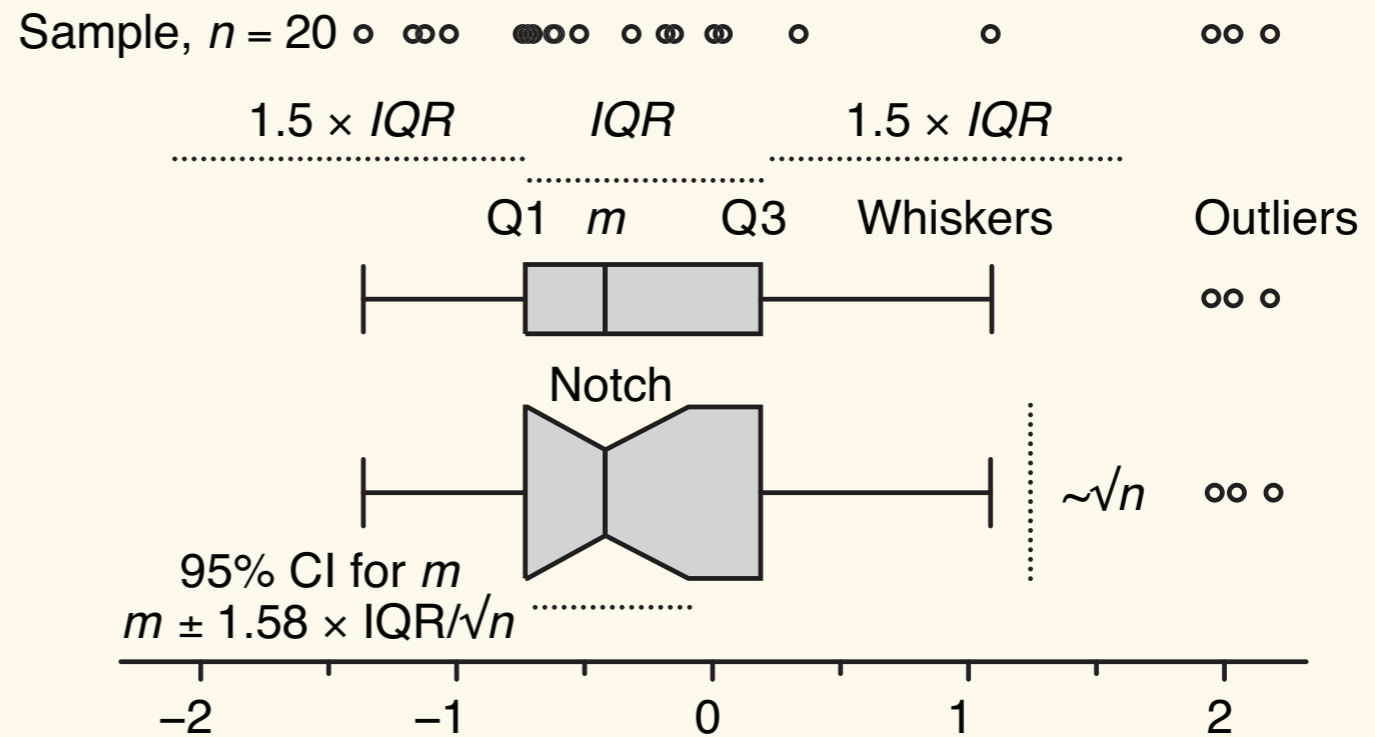
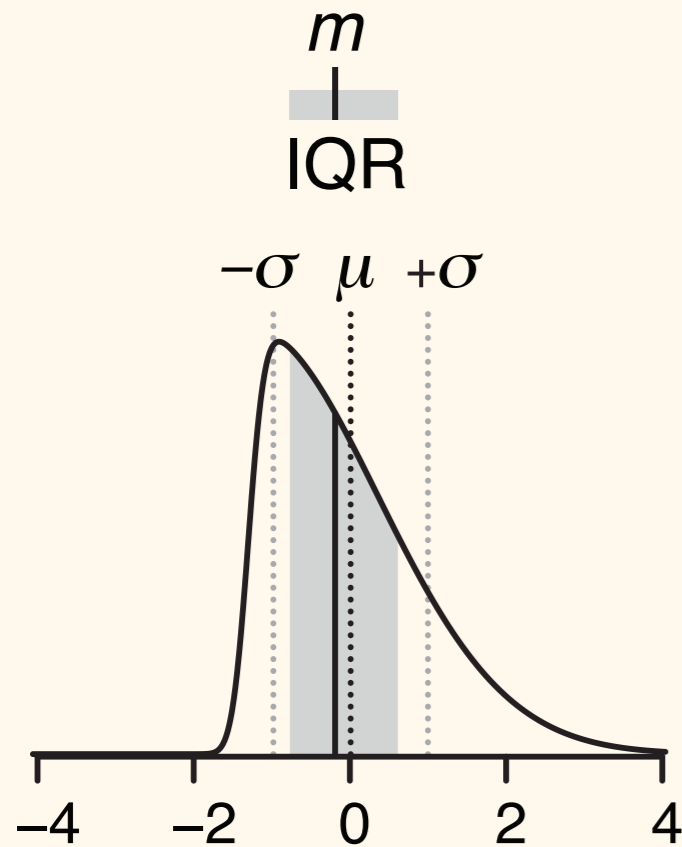
Population distribution





# Boxplots (also invented by Tukey)

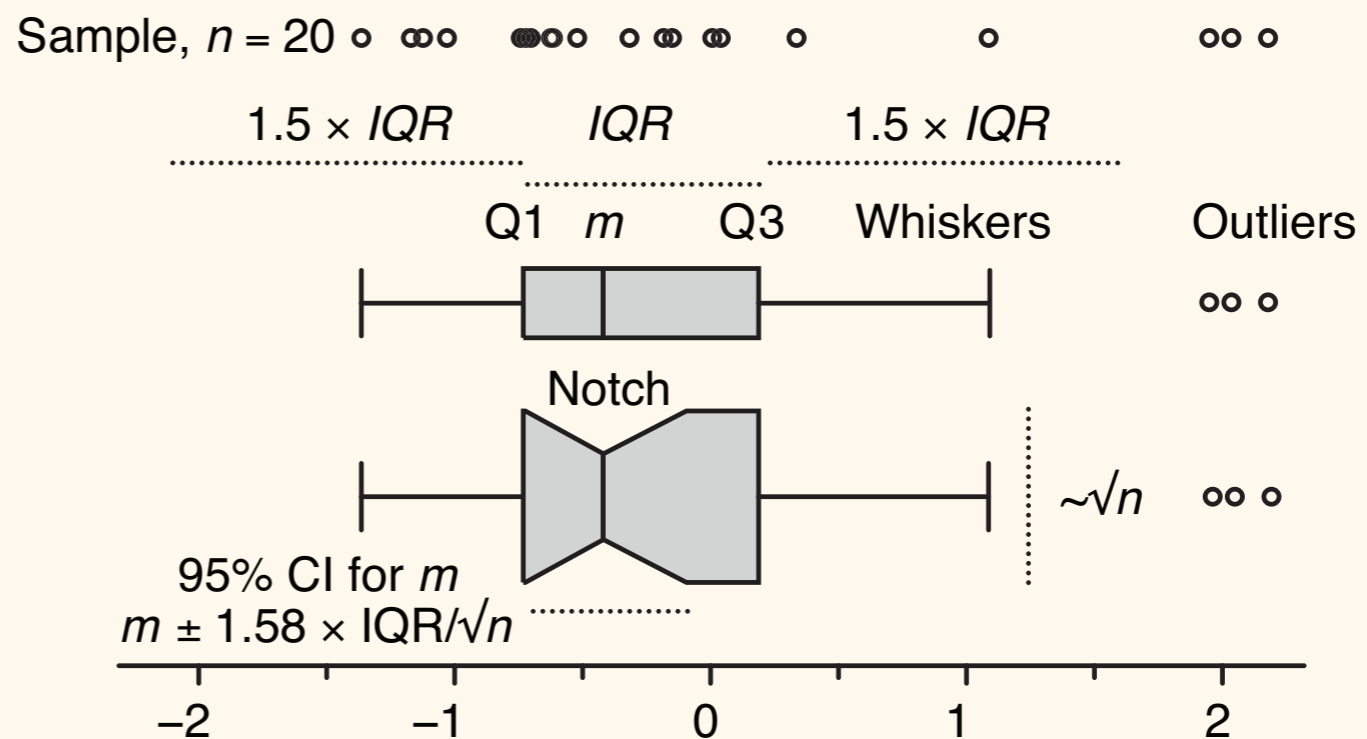
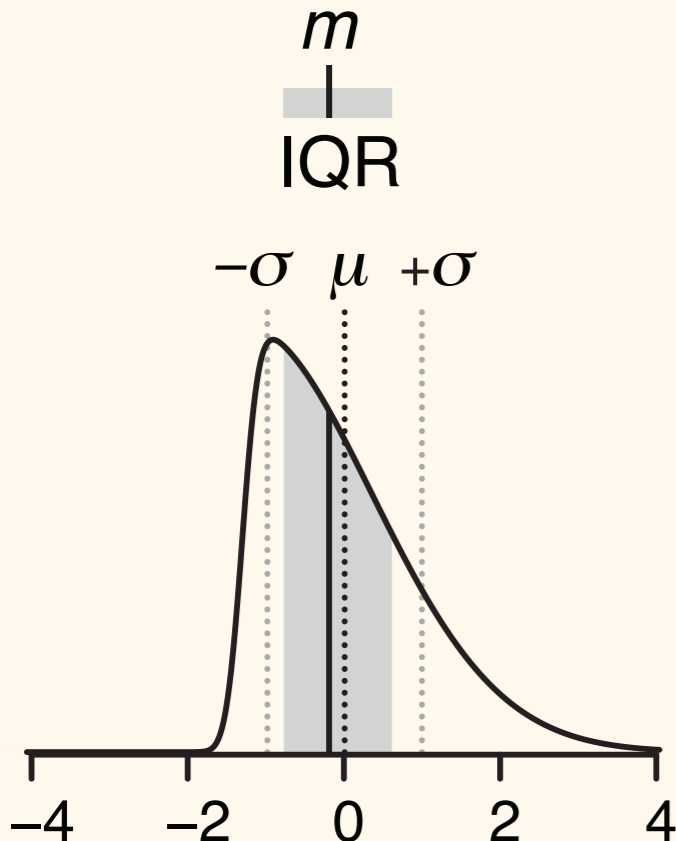
## Population distribution



**Figure 1** | The construction of a box plot. (a) The median ( $m = -0.19$ , solid vertical line) and interquartile range ( $IQR = 1.38$ , gray shading) are ideal for characterizing asymmetric or irregularly shaped distributions. A skewed normal distribution is shown with mean  $\mu = 0$  (dark dotted line) and s.d.  $\sigma = 1$  (light dotted lines). (b) Box plots for an  $n = 20$  sample from a. The box bounds the IQR divided by the median, and Tukey-style whiskers extend to a maximum of  $1.5 \times IQR$  beyond the box. The box width may be scaled by  $\sqrt{n}$ , and a notch may be added approximating a 95% confidence interval (CI) for the median. Open circles are sample data points. Dotted lines indicate the lengths or widths of annotated features.

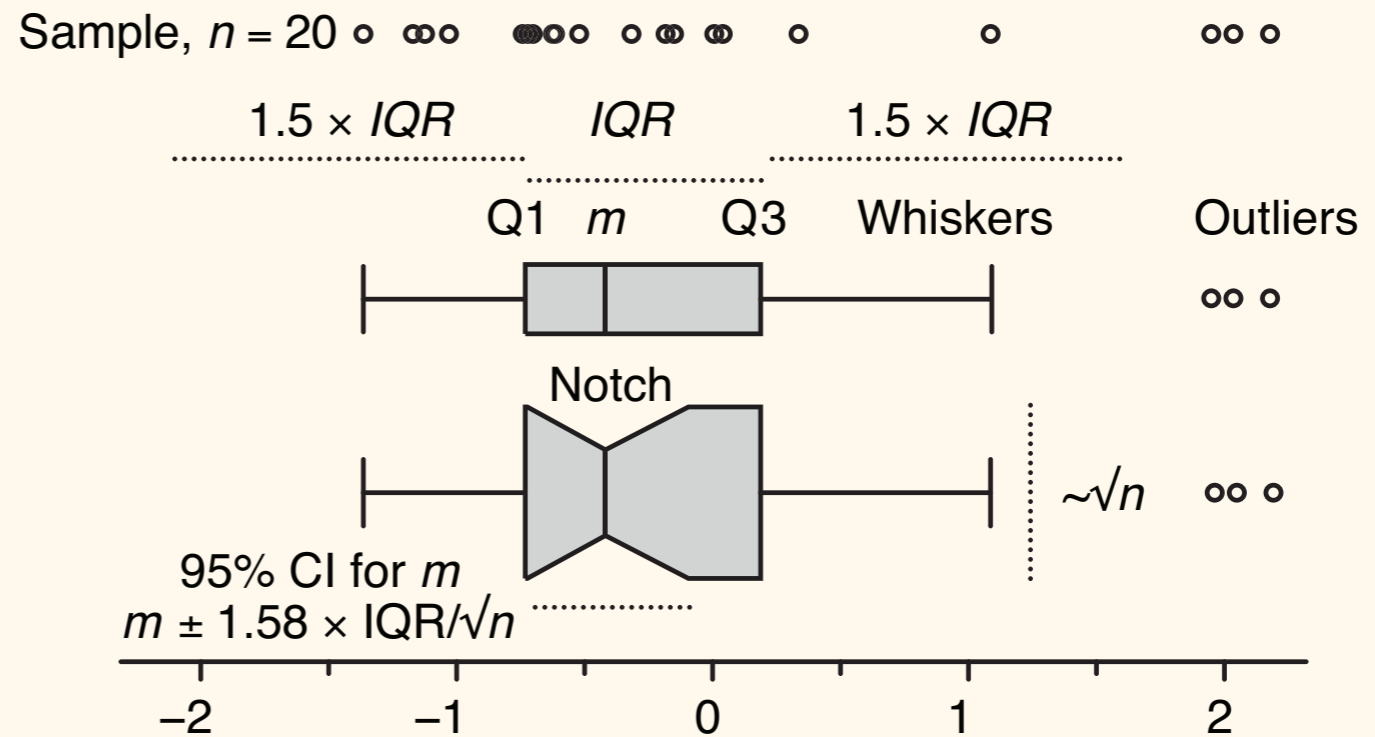
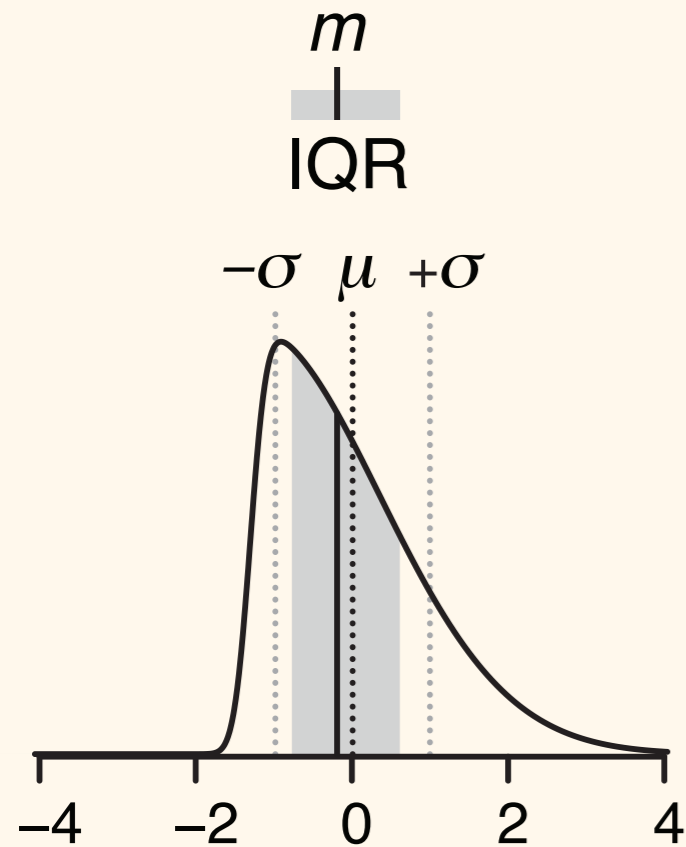
# Boxplots (also invented by Tukey)

Population distribution



# Boxplots (also invented by Tukey)

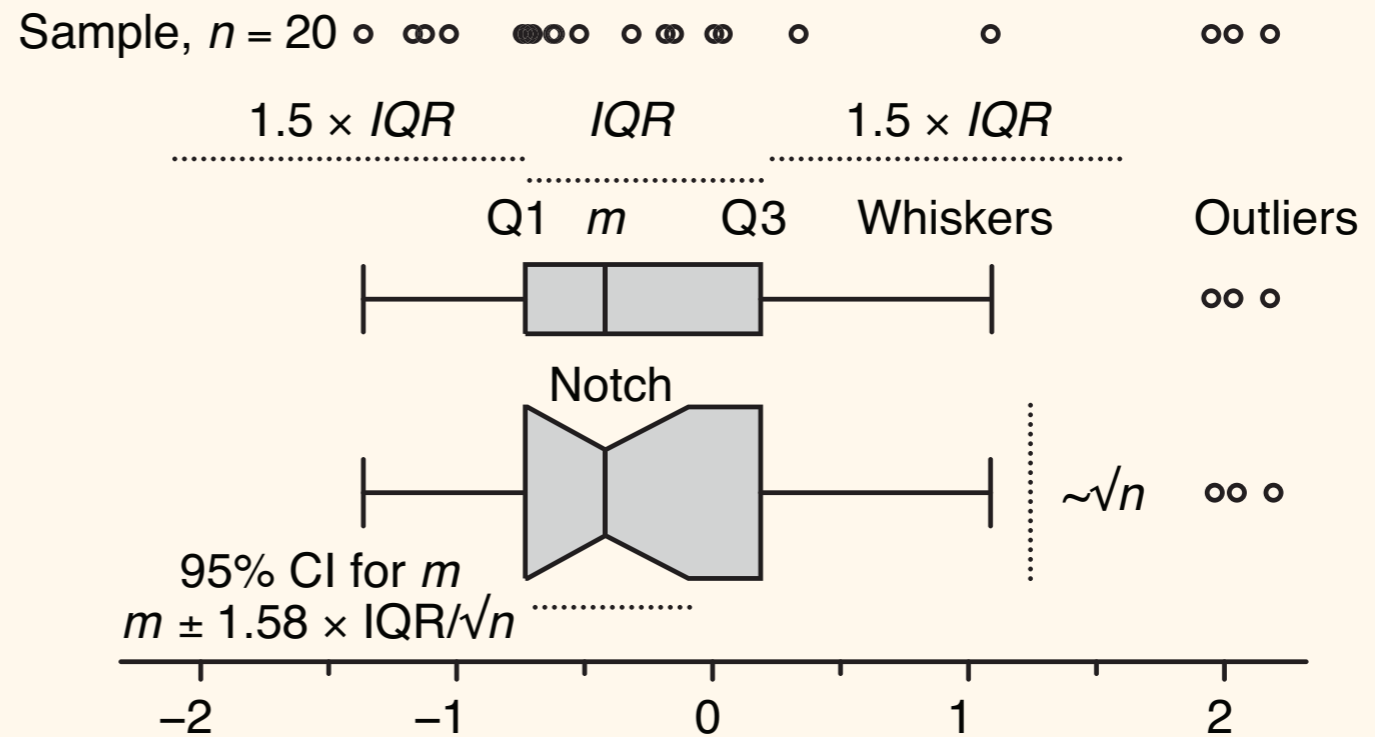
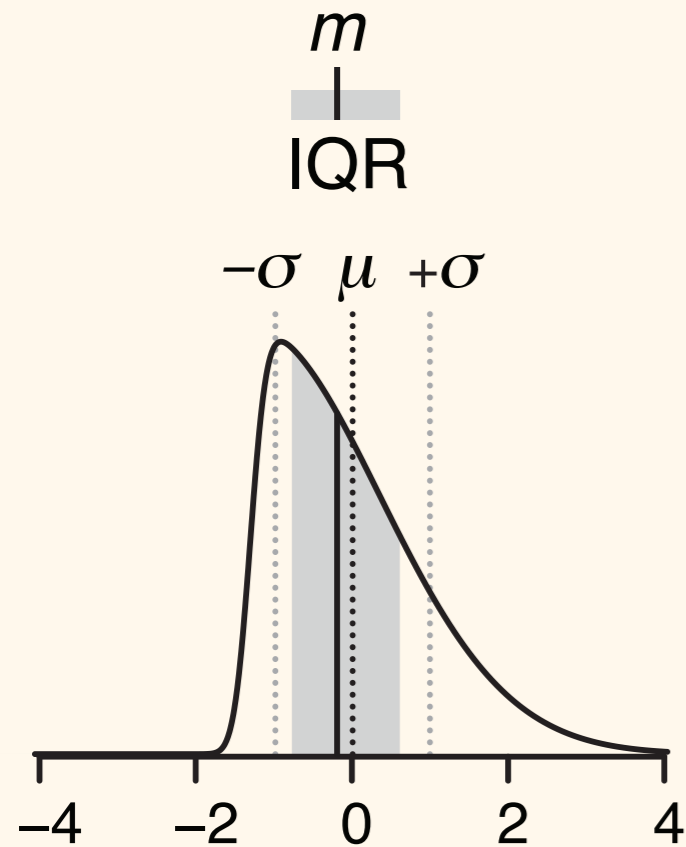
Population distribution



Tukey-style whiskers: the most extreme data point that is no more than  $1.5 \times IQR$  from the edge of the box...

# Boxplots (also invented by Tukey)

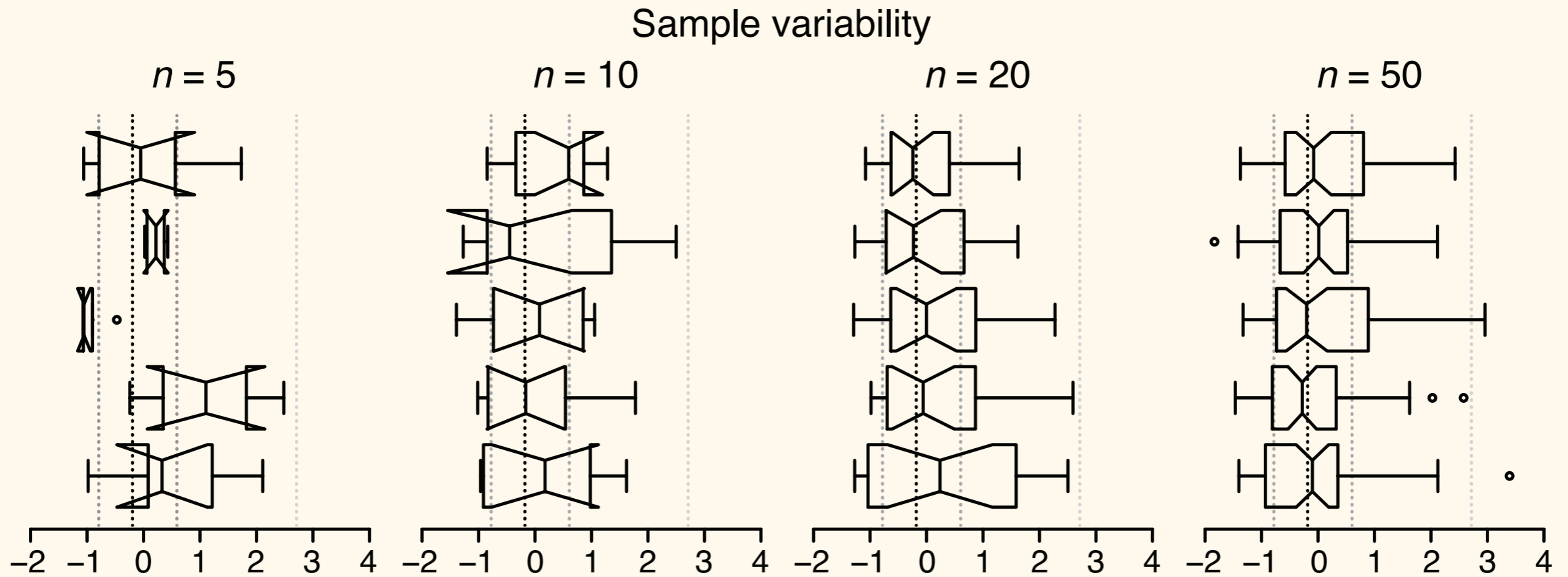
Population distribution



Tukey-style whiskers: the most extreme data point that is no more than  $1.5 \times IQR$  from the edge of the box...

Spear-style whiskers: the most extreme values, period (min/max).

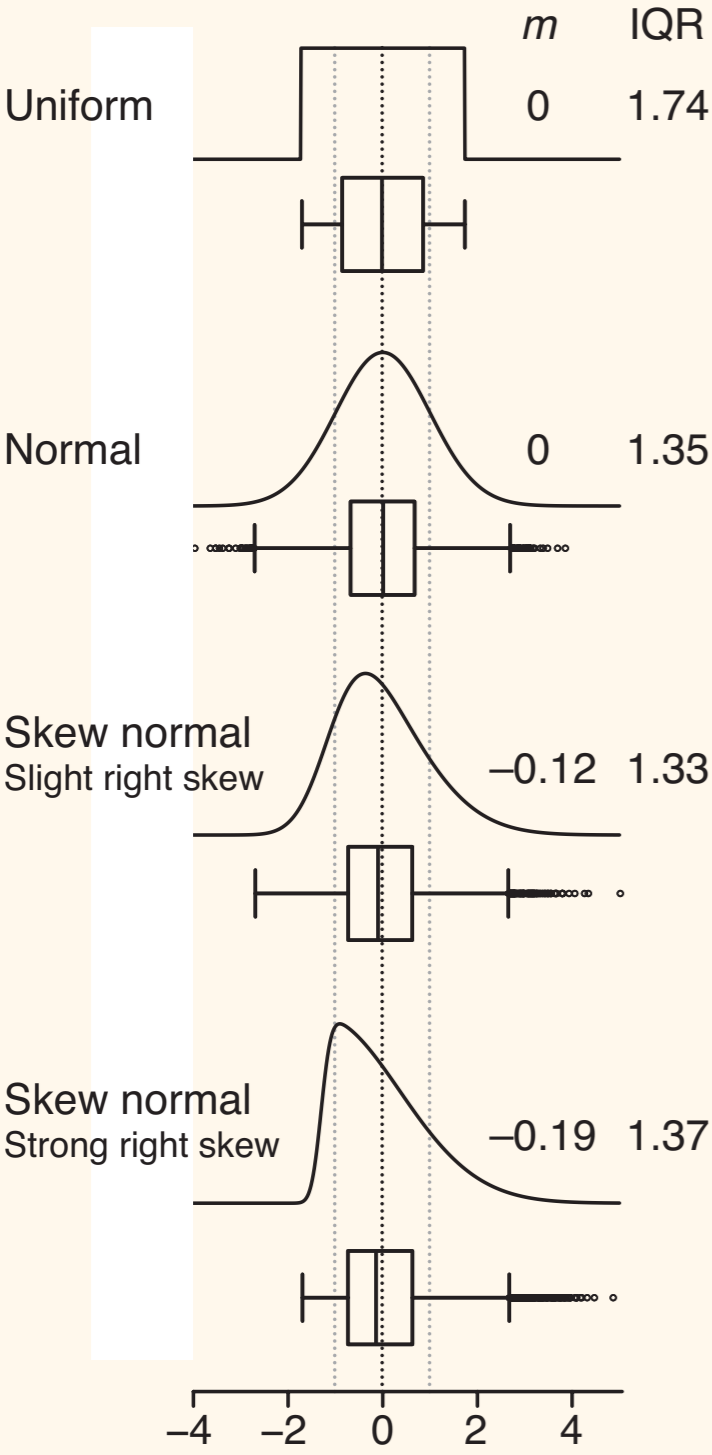
# Boxplots (also invented by Tukey)



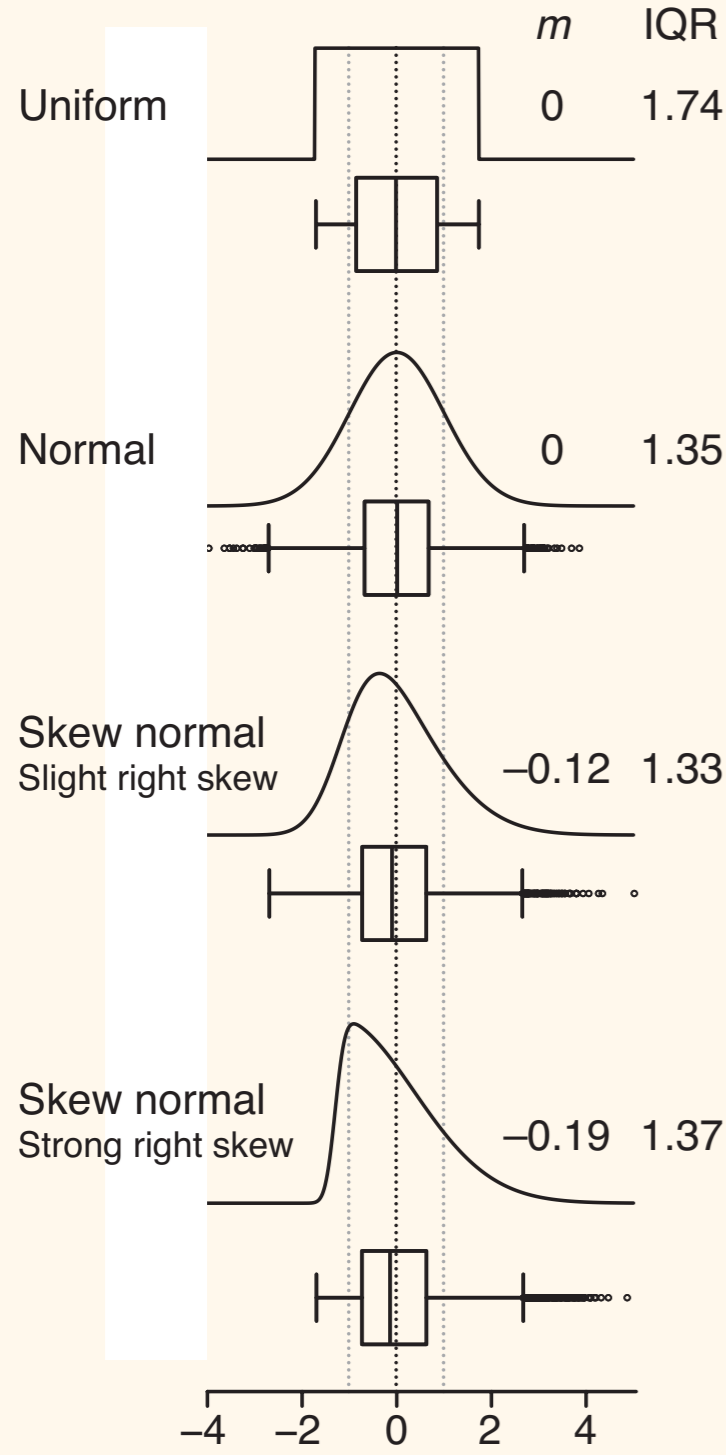
Aspects of the box plot such as width, whisker position, notch size and outlier display are subject to tuning; it is therefore important to clearly label how your box plot was constructed. Fewer than 20% of box plot figures in 2013 *Nature Methods* papers specified both sample size and whisker type in their legends—we encourage authors to be more specific.

# Boxplots (also invented by Tukey)

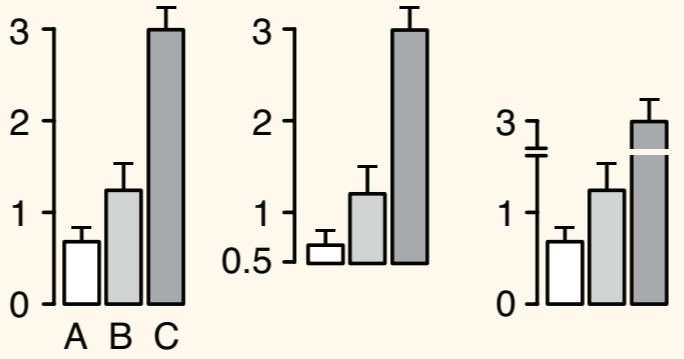
# Boxplots (also invented by Tukey)



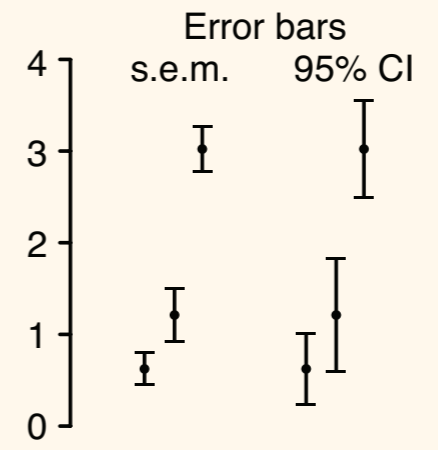
# Boxplots (also invented by Tukey)



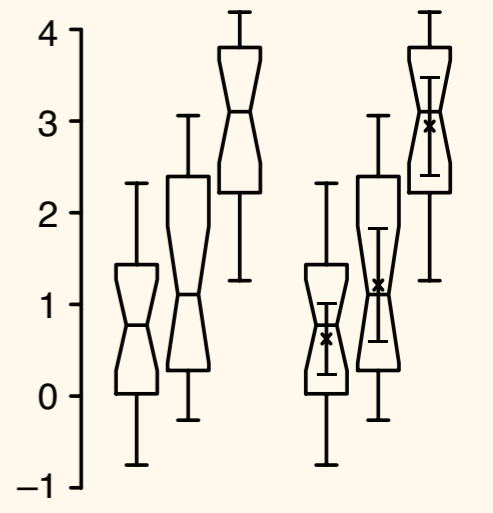
**a** Means as bar plots  
Not recommended



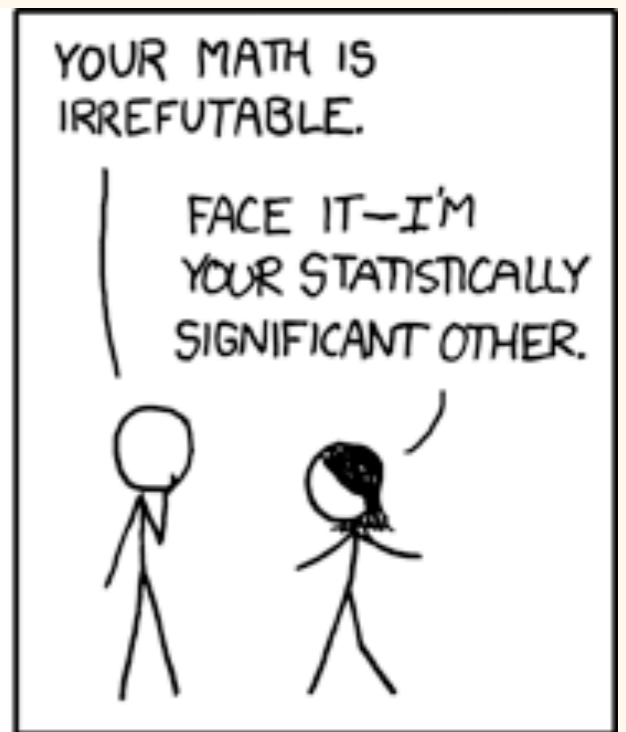
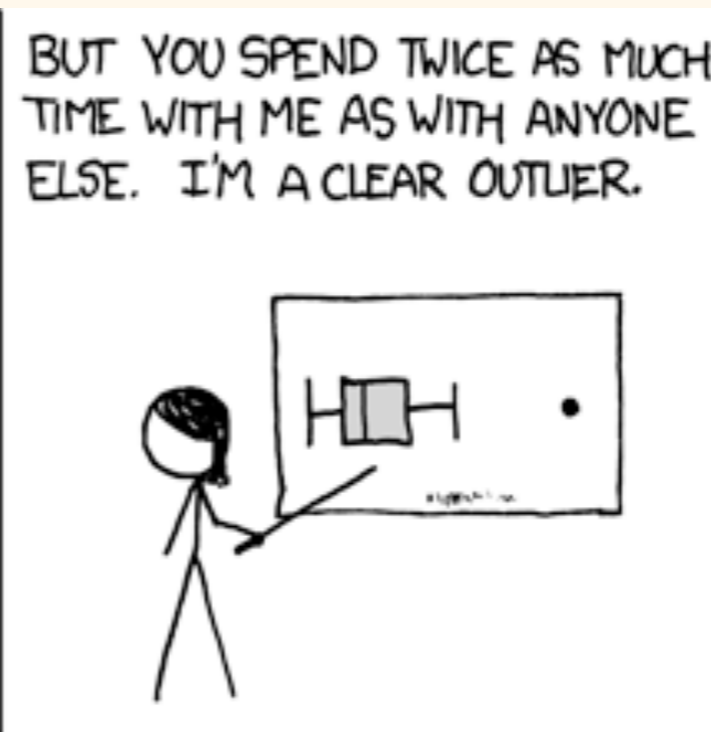
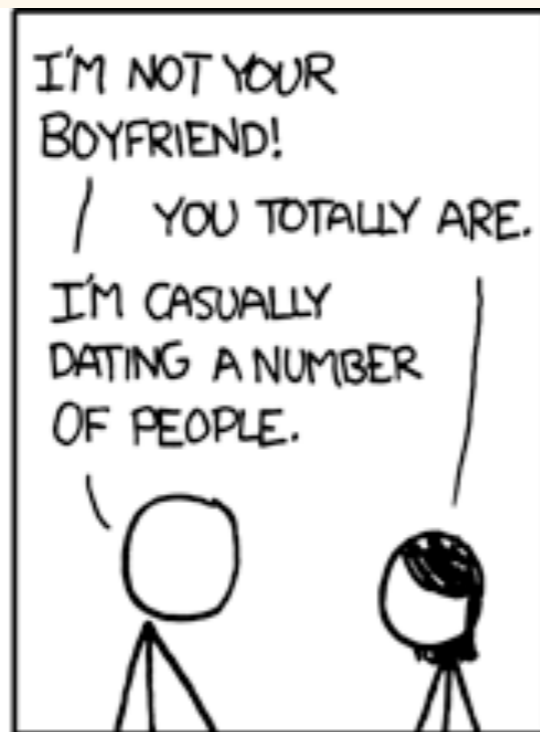
**b** Means as scatter plots  
Error bars



**c** Box plots with optional means and 95% CI







# Related (but not invented by Tukey):

## Statistical Computing and Graphics

### Violin Plots: A Box Plot-Density Trace Synergism

Jerry L. HINTZE and Ray D. NELSON

Many modifications build on Tukey's original box plot. A proposed further adaptation, the violin plot, pools the best statistical features of alternative graphical representations of batches of data. It adds the information available from local density estimates to the basic summary statistics inherent in box plots. This marriage of summary statistics and density shape into a single plot provides a useful tool for data analysis and exploration.

**KEY WORDS:** Density estimation; Exploratory data analysis; Graphical techniques.

#### 1. INTRODUCTION

Many different statistics and graphs summarize the characteristics of single batches of data. Descriptive statistics give information about location, scale, symmetry, and tail thickness. Other statistics and graphs investigate extreme observations or study the distribution of data values. Diagrams such as stem-leaf plots, dot plots, box plots, histograms, density traces, and probability plots give information about the distribution of values assumed by all observations.

The violin plot, introduced in this article, synergistically combines the box plot and the density trace (or smoothed histogram) into a single display that reveals structure found within the data. The introduction of this new graphical tool begins with a quick overview of the combination of the box plot and density trace into the violin plot. Then, three illustrations and examples show the advantages and challenges of violin plots in data summarization and exploration.

#### 2. COMPONENT PARTS OF VIOLIN PLOTS

The violin plot, as depicted in Figure 1 and implemented in NCSS (1997) statistical software, combines the box plot and density trace into one diagram. The name *violin plot* originated because one of the first analyses that used the envisioned procedure resulted in a graphic with the appearance of a violin. Violin plots add information to the simple structure of the box plot that Tukey (1977) initially conceived. Although these original graphs are easily drawn with pencil and paper, computers ease subsequent modifications, refinements, and computation of box plots as discussed by McGill, Tukey, and Larsen (1978); Velleman and

Hoaglin (1981); Chambers, Cleveland, Kleiner, and Tukey (1983); Frigge, Hoaglin, and Iglewicz (1989), and others.

Box plots show four main features about a variable: center, spread, asymmetry, and outliers. As an example, consider the box plot in Figure 1 for the data published by Hamermesh (1994). The ASA Statistical Graphics Section's 1995 Data Analysis Exposition analyzes these data, which report compensation of professors from all academic ranks in the United States. The labels in the diagram identify the principal lines and points which form the main structure of the traditional box plot diagram. As shown, the violin plot includes a box plot with two slight modifications. First, a circle replaces the median line which facilitates quick comparisons when viewing multiple groups. Second, outside points which are traditionally classified as mild and severe outliers, are not identified by individual symbols.

The density trace supplements traditional summary statistics by graphically showing the distributional characteristics of batches of data. One simple density estimator, the histogram, displays the distribution of data values along the real number line. Weaknesses of the histogram caused Tapia and Thompson (1979), Parzen (1979), Silverman (1986), Izenman (1991), and Scott (1992) to propose and summarize numerous alternative density estimators. One of these alternatives is the density trace described in Chambers, Cleveland, Kleiner, and Tukey (1983). Defining the location density  $d(x|h)$  at a point  $x$  as the fraction of the data values per unit of measurement that fall in an interval centered at  $x$  gives

$$d(x|h) = \frac{\sum_{i=1}^n \delta_i}{nh}, \quad (1)$$

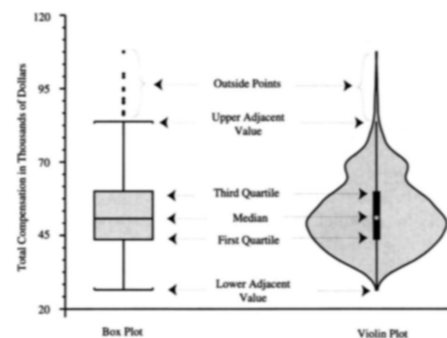


Figure 1. Common Components of Box Plot and Violin Plot. Total compensation for all academic ranks.

Jerry L. Hintze is President, NCSS, 329 North 1000 East, Kaysville, UT 84037 (E-mail: sales@ncss.com). Ray D. Nelson is Associate Professor of Business Management, Marriott School of Management, Brigham Young University, Provo, UT 84602.

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The American Statistician, May 1998 Vol. 52, No. 2 181

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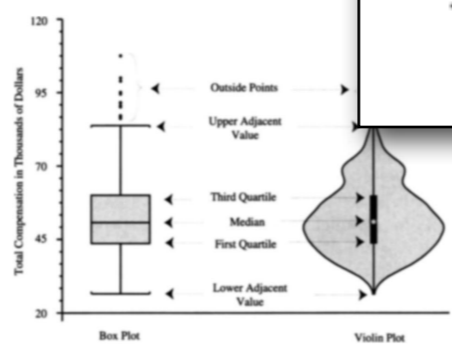


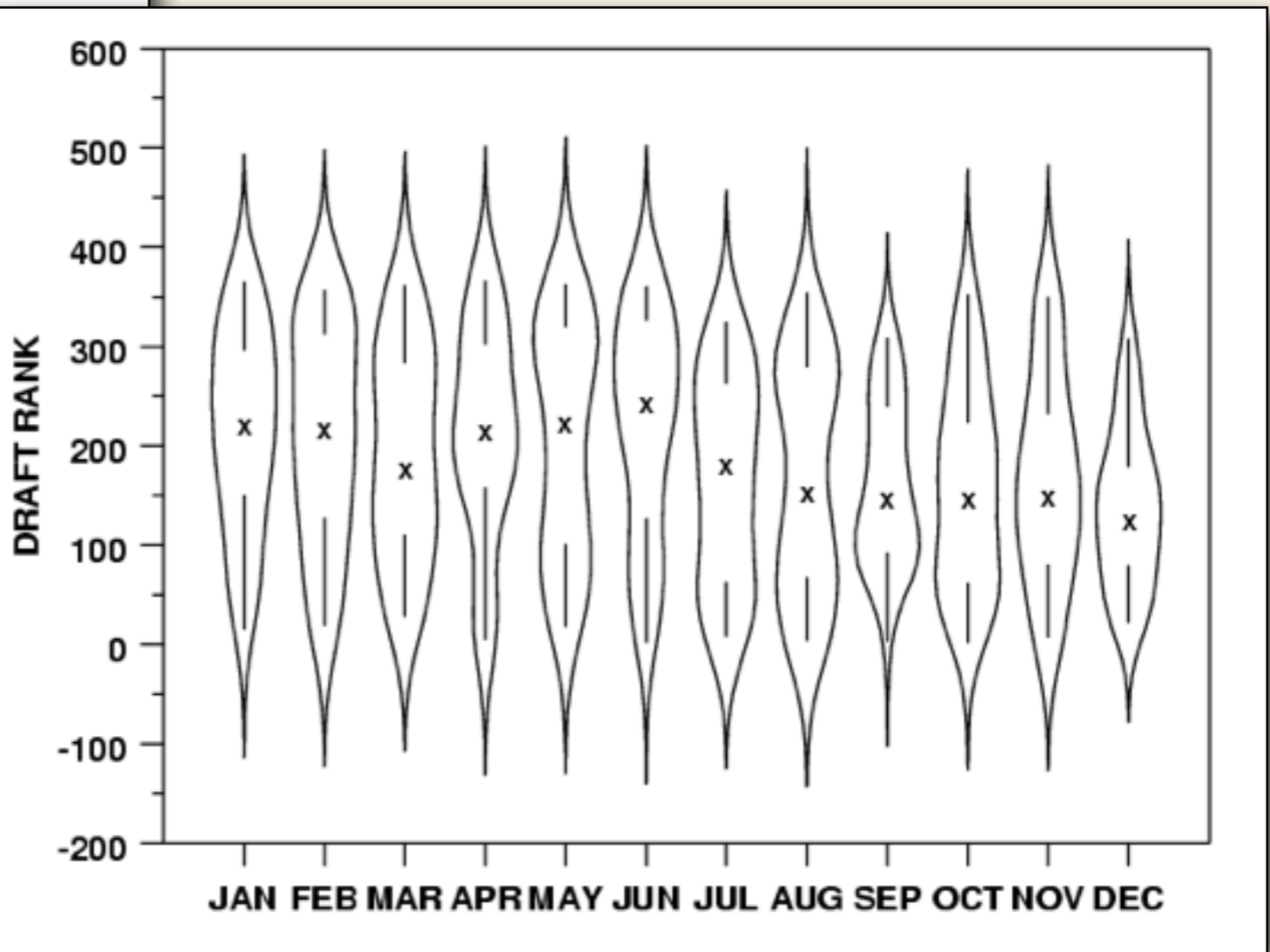
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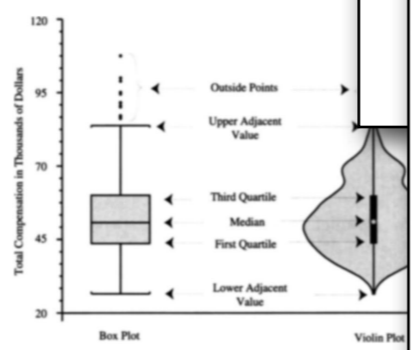


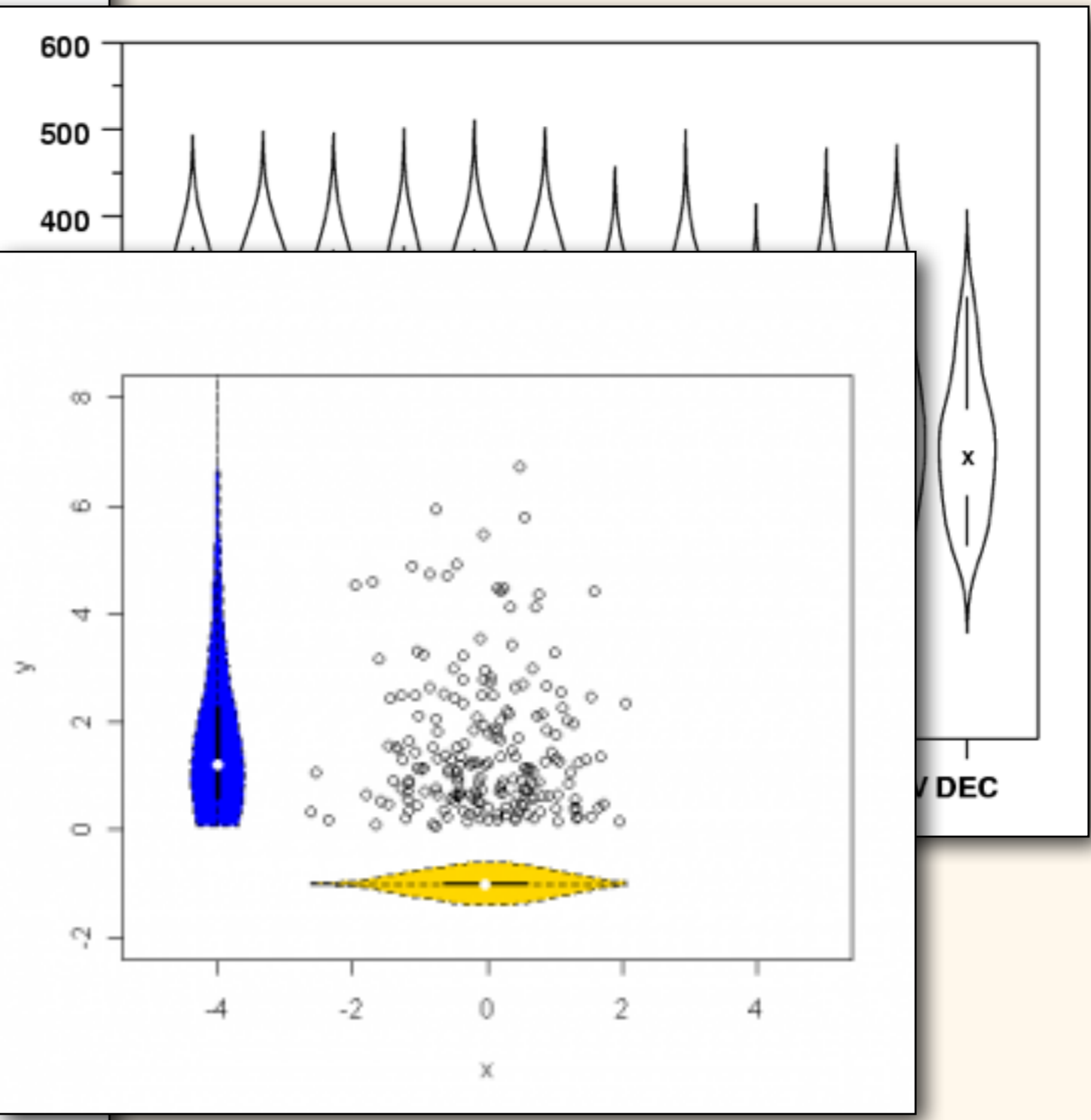
Figure 1. Common Components of Box Plot and Violin Plot for compensation for all academic ranks.

The American Statistician, May 1998 Vol. 52, No. 2

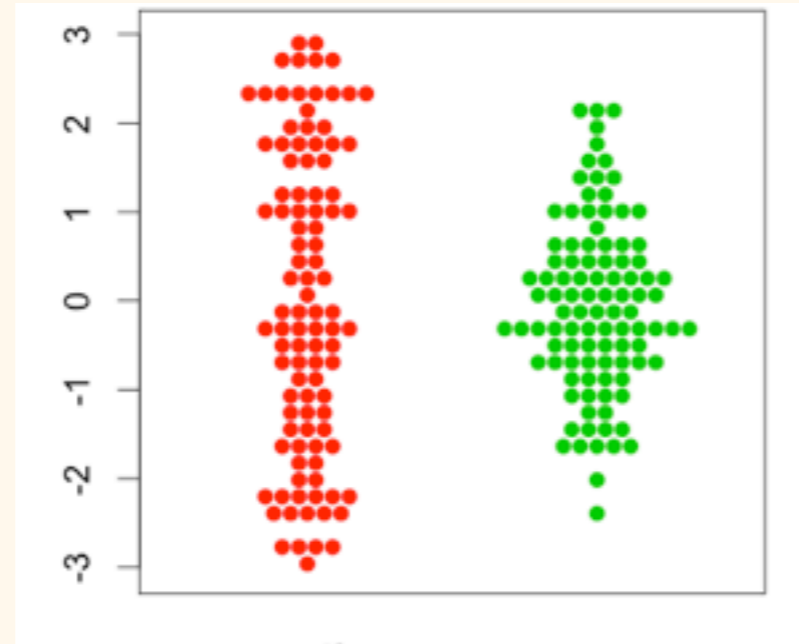
Jerry L. Hintze is President, NCSS, 329 North 1000 East, Kaysville, UT 84037 (E-mail: sales@ncss.com). Ray D. Nelson is Associate Professor of Business Management, Marriott School of Management, Brigham Young University, Provo, UT 84602.

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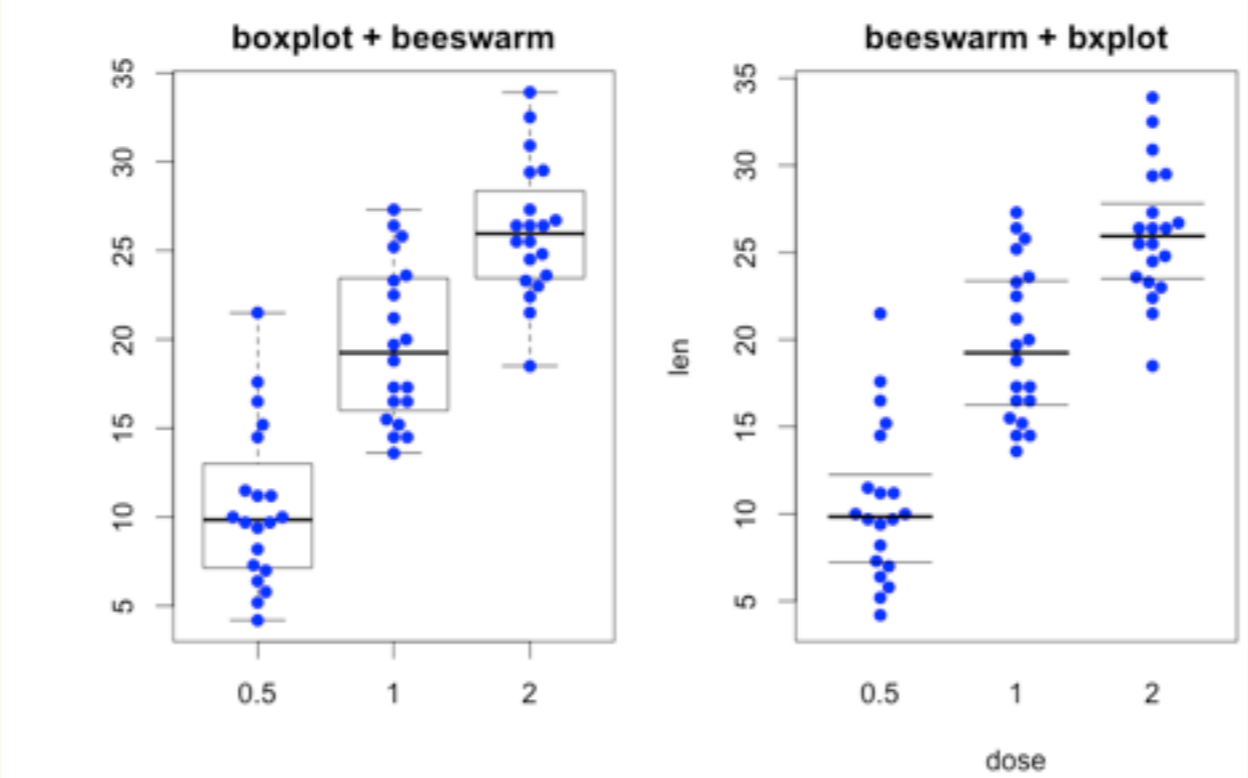
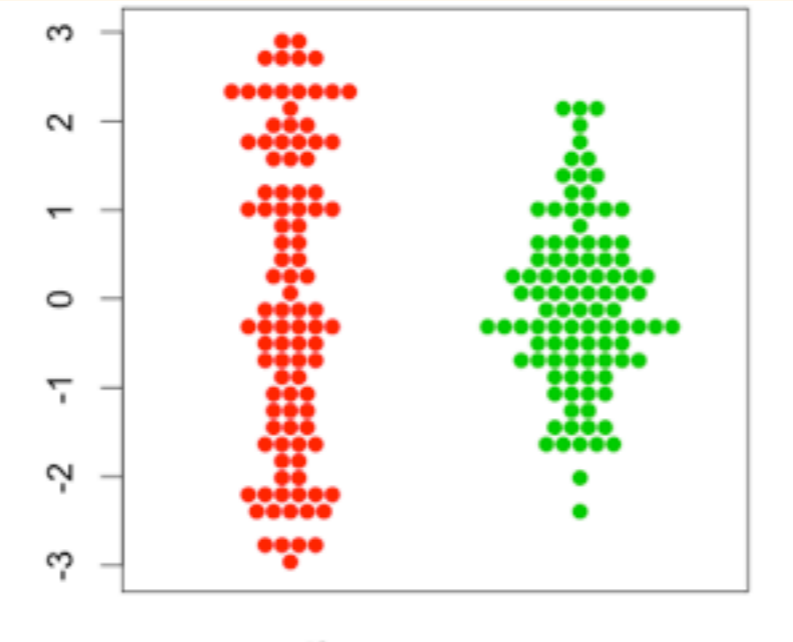
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# Beeswarms:



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# Other examples:

<b>Revenue</b>													
Region	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
West	28,384	30,288	34,302	32,039	32,938	34,392	33,923	33,092	34,934	30,384	33,923	37,834	396,433
Central	15,934	16,934	17,173	16,394	17,345	16,384	15,302	14,939	14,039	12,304	11,033	9,283	177,064
East	11,293	12,384	12,938	12,034	11,034	13,983	12,384	12,374	12,384	13,374	14,394	19,283	157,859
Total	55,611	59,606	64,413	60,467	61,317	64,759	61,609	60,405	61,357	56,062	59,350	66,400	731,356

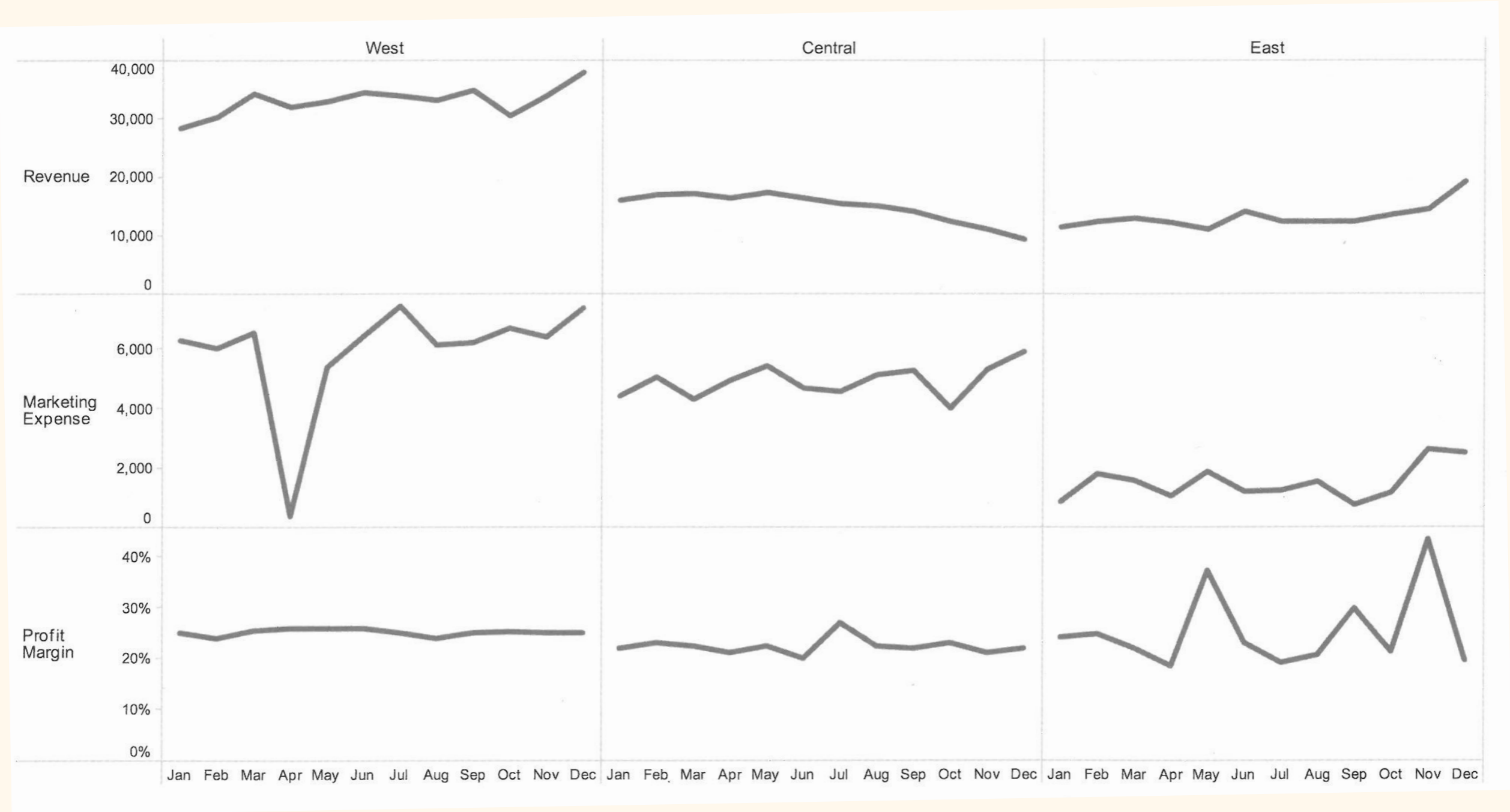
## **Marketing Expenses**

Region	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
West	6,288	6,019	6,555	364	5,407	6,450	7,442	6,150	6,201	6,697	6,408	7,376	71,356
Central	4,429	5,039	4,309	4,951	5,442	4,675	4,558	5,124	5,278	4,016	5,325	5,898	59,044
East	851	1,784	1,542	1,024	1,864	1,173	1,237	1,504	714	1,152	2,620	2,501	17,966
Total	11,568	12,842	12,406	6,339	12,713	12,298	13,237	12,778	12,192	11,865	14,353	15,775	148,367

## **Profit Margin**

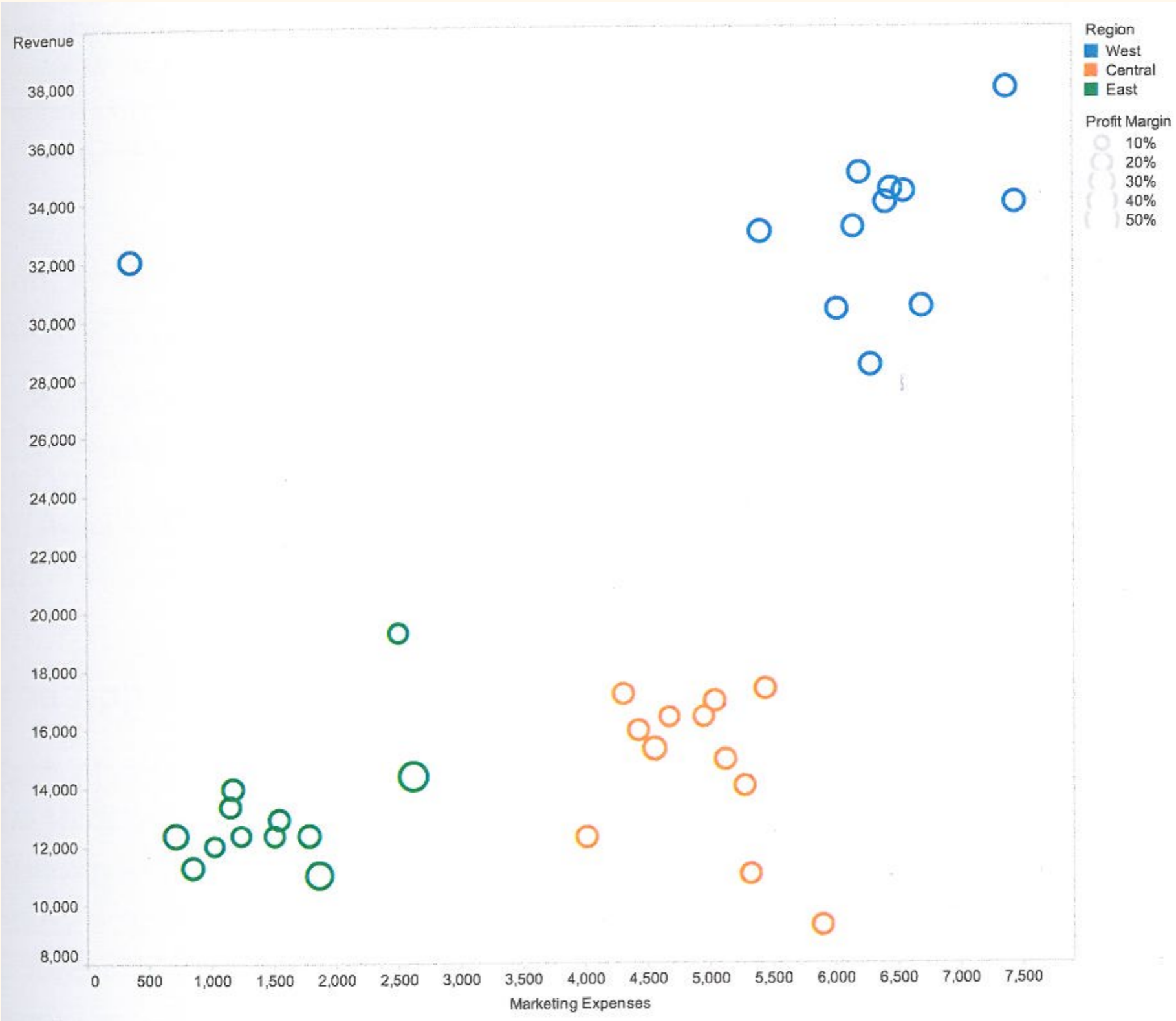
Region	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Average
West	25.11%	24.07%	25.52%	25.80%	25.93%	26.06%	25.02%	24.41%	25.13%	25.31%	25.12%	25.01%	25.13%
Central	22.13%	23.22%	22.55%	21.08%	22.54%	20.04%	27.08%	22.52%	22.31%	23.32%	21.05%	22.01%	22.38%
East	24.06%	24.80%	21.97%	18.50%	37.16%	23.02%	19.06%	20.60%	29.74%	21.41%	43.29%	19.49%	25.26%
Average	23.69%	23.93%	23.32%	21.77%	28.52%	23.01%	23.69%	22.37%	25.58%	23.24%	29.80%	22.16%	24.26%

# Other examples:

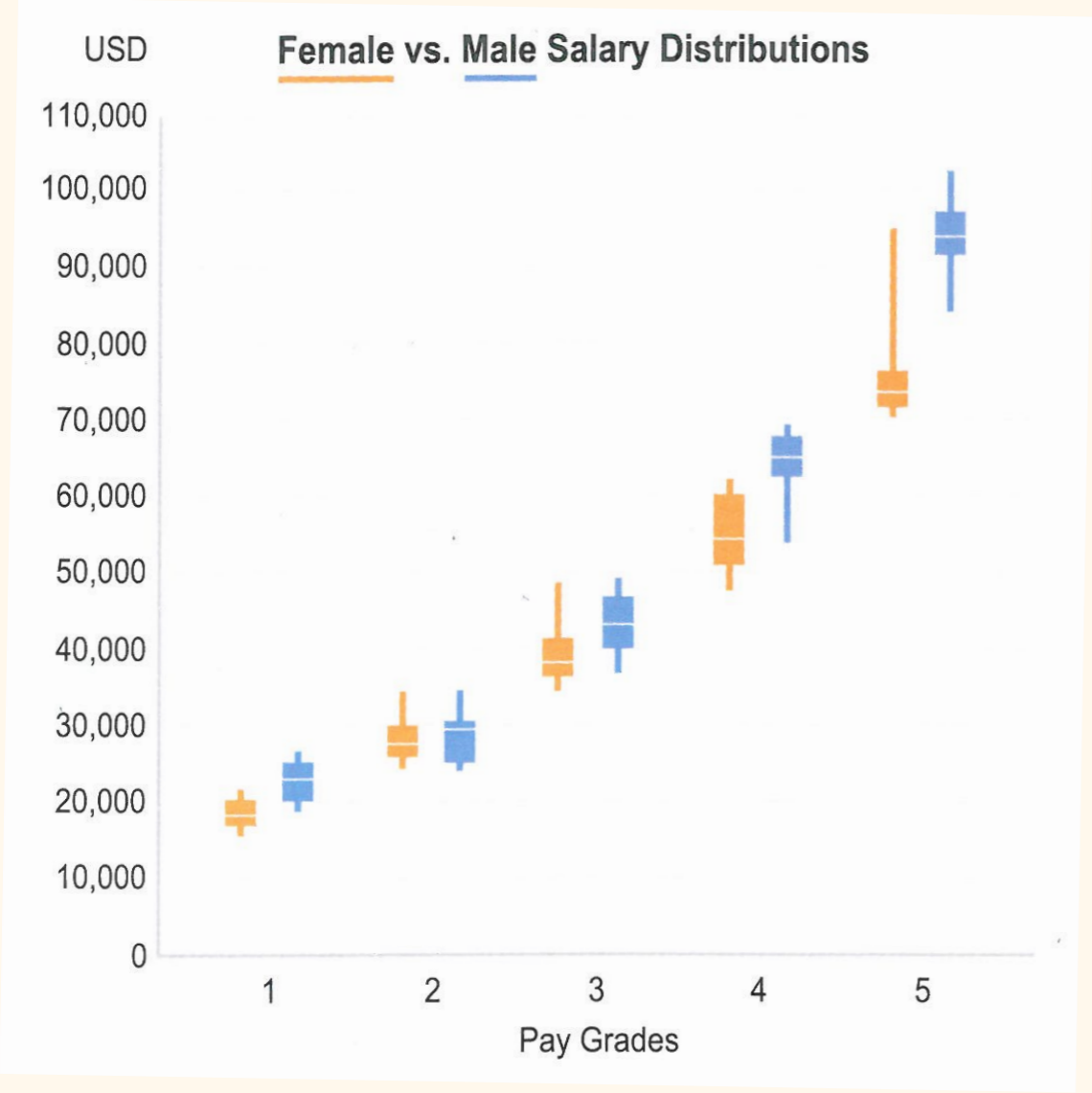




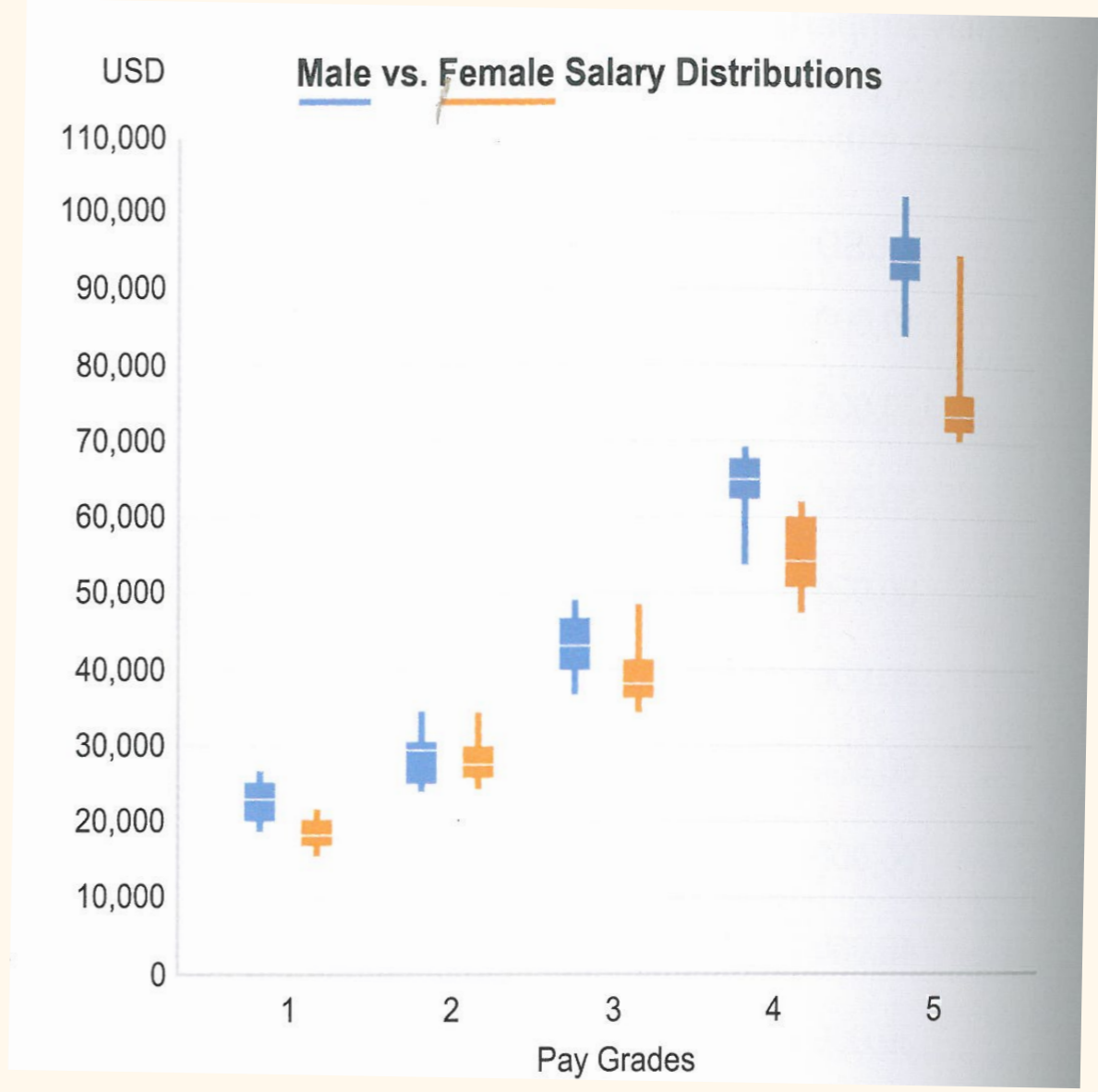
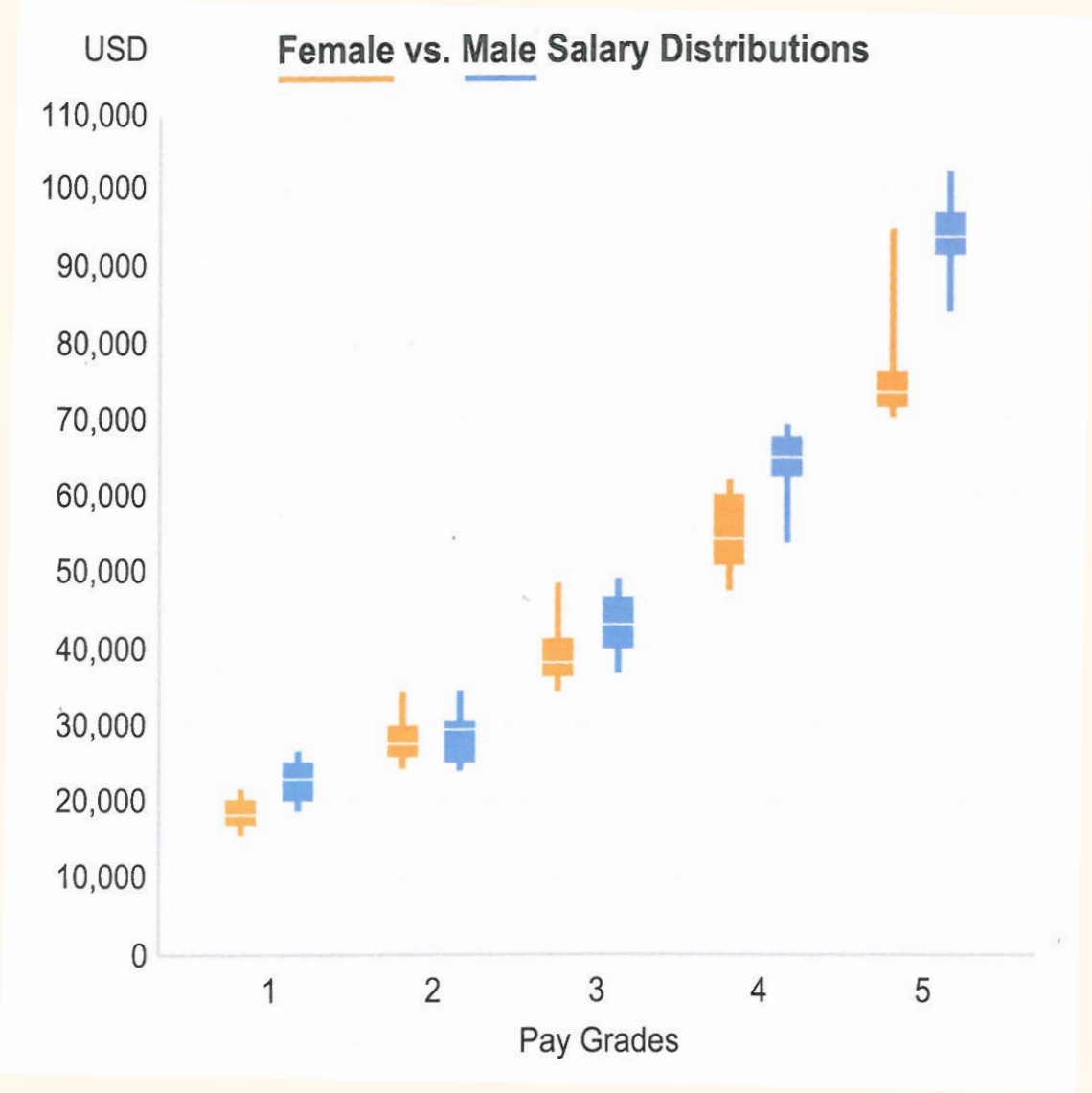
# Other examples:



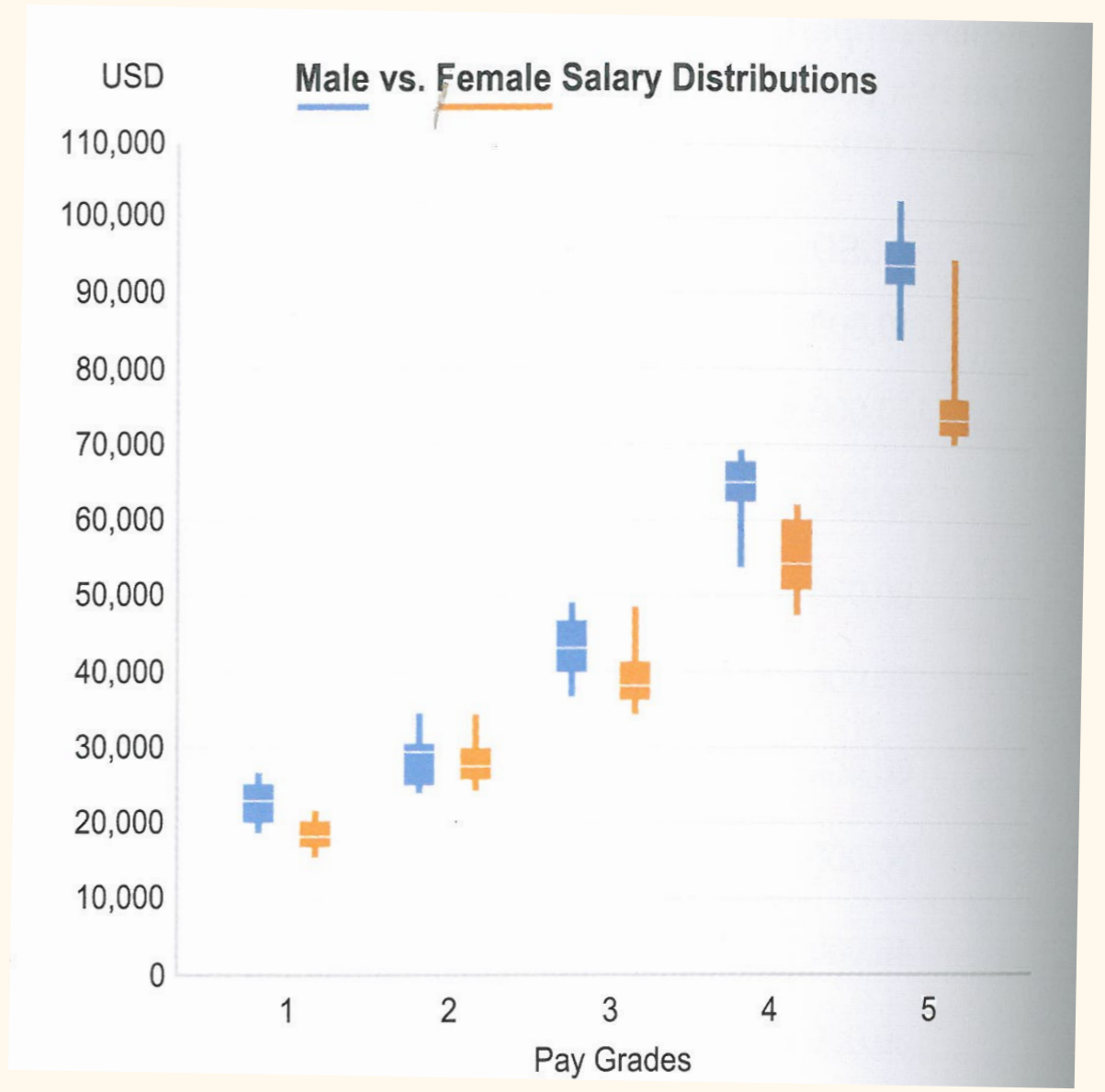
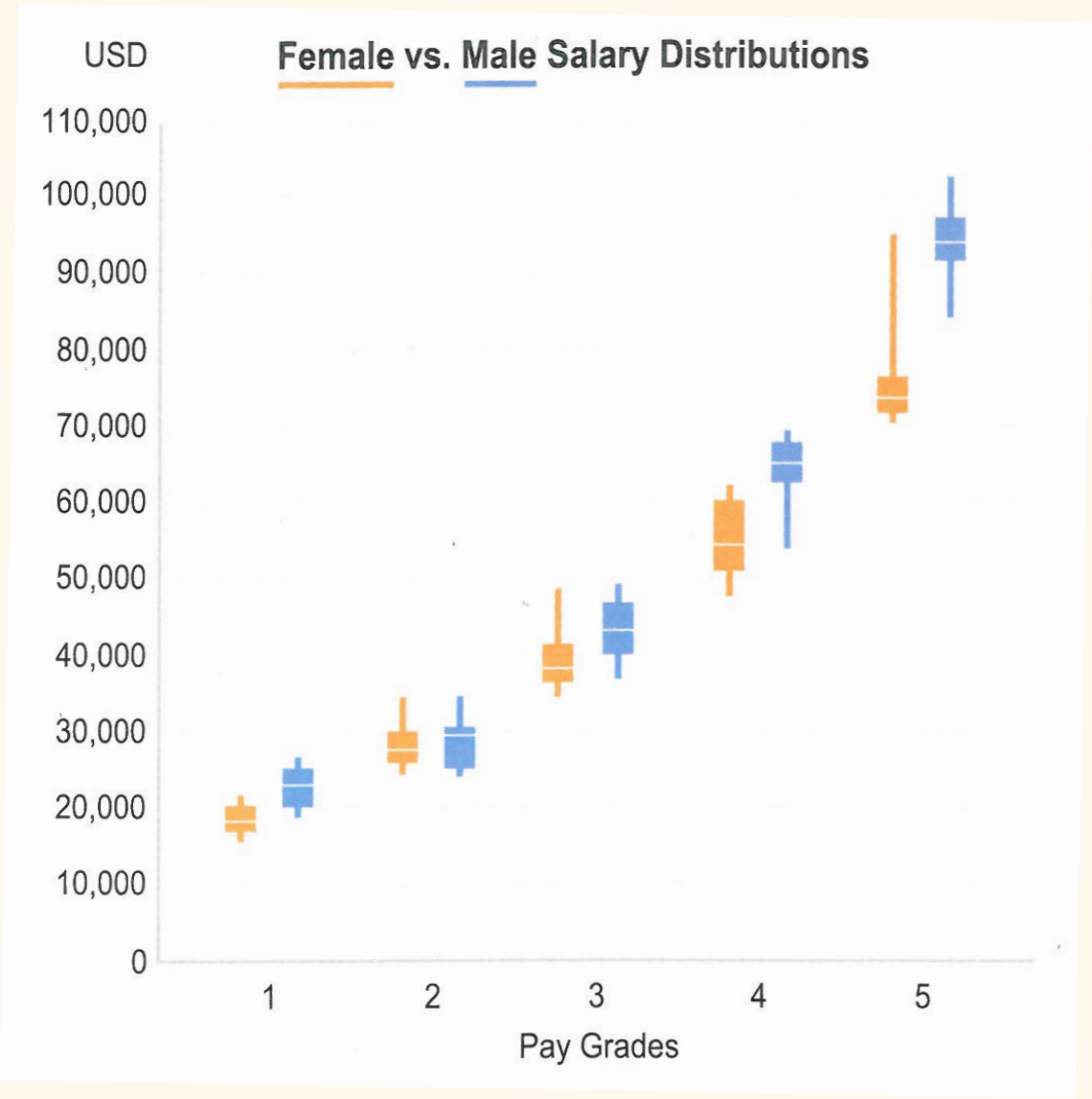
# Other examples:



# Other examples:



# Other examples:



The Gestalt principle of continuation is making the graph on the left look like a much smoother curve than it really is...

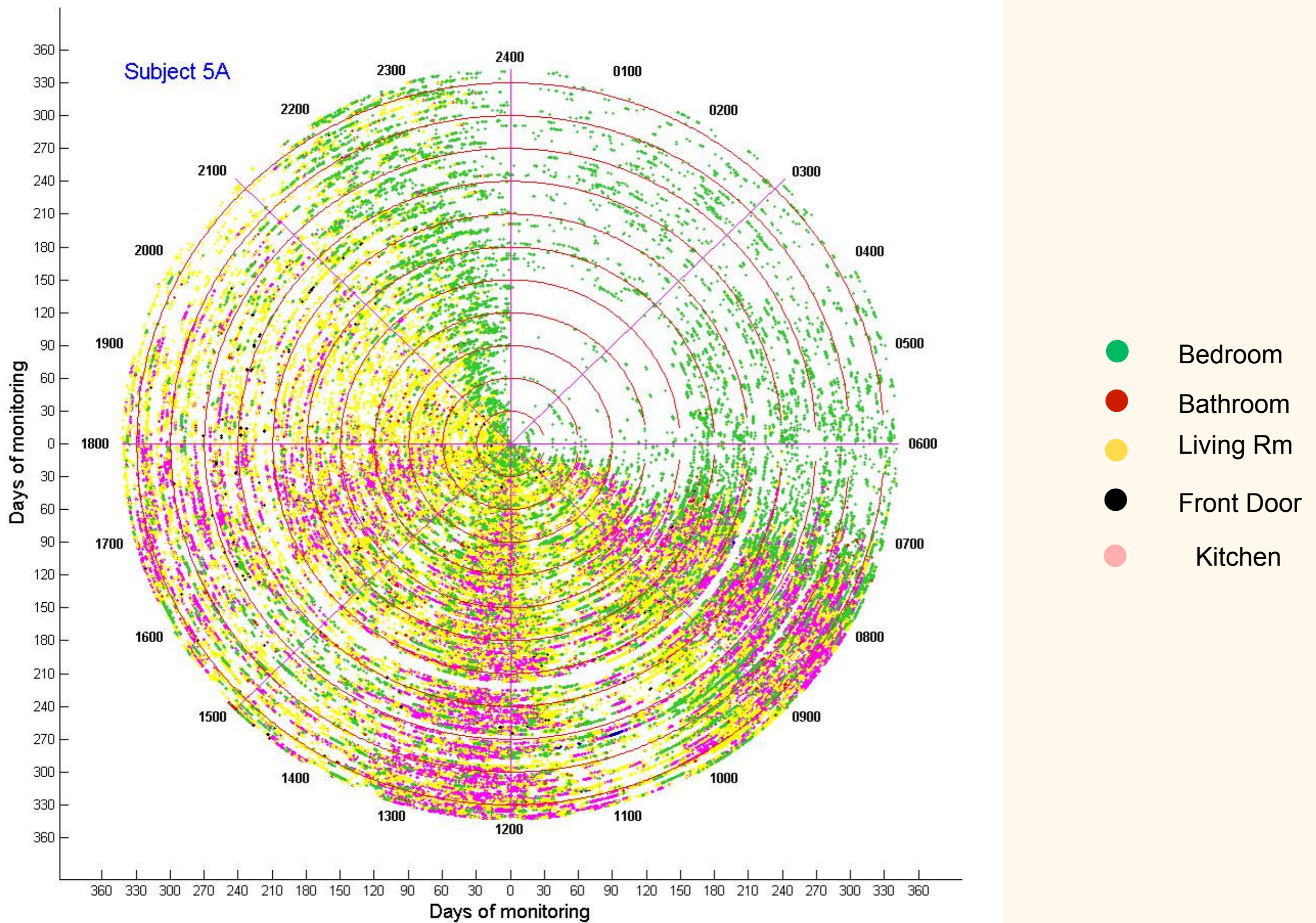


Figure courtesy Holly Jimison, Tamara Hayes

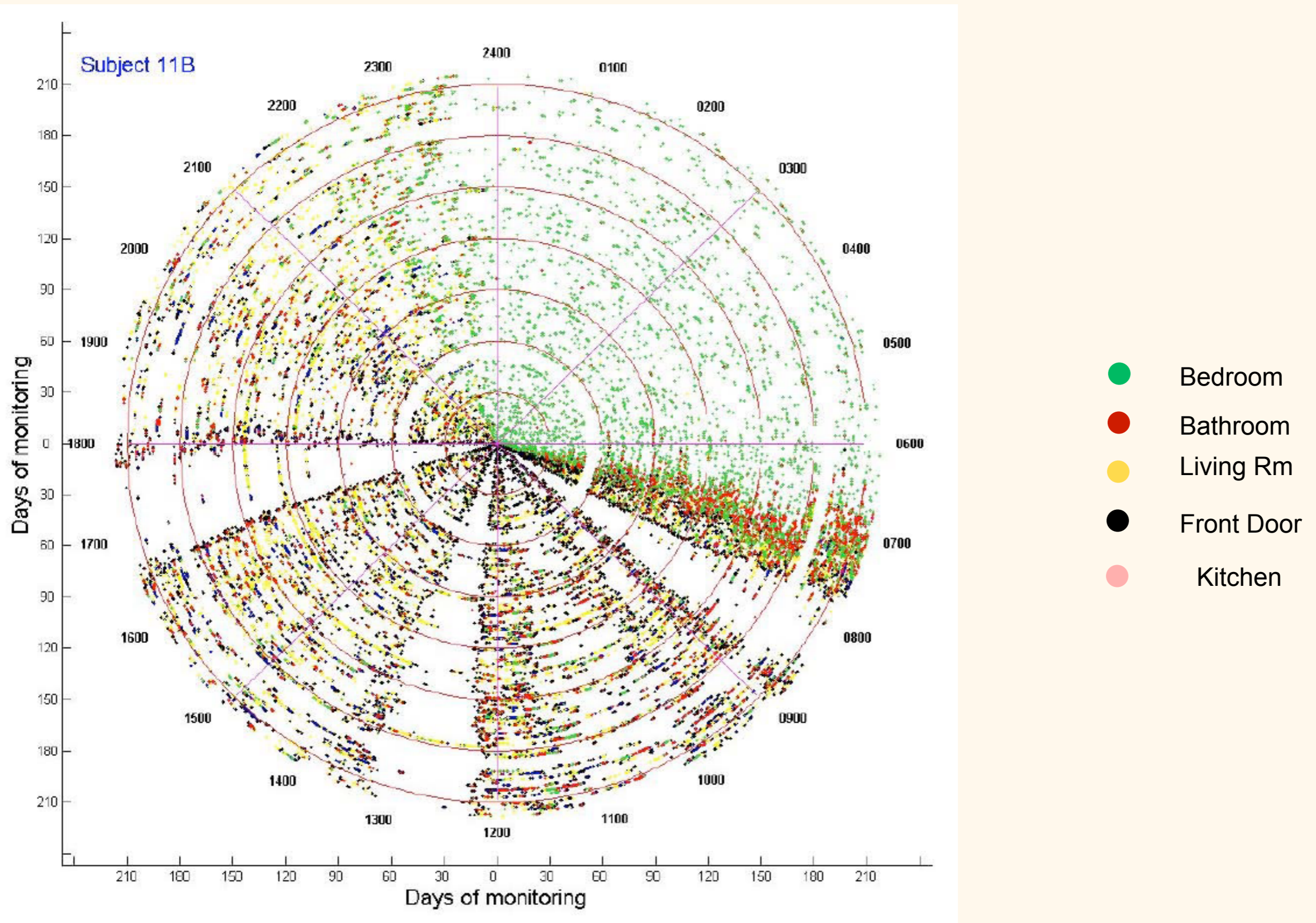


Figure courtesy Holly Jimison, Tamara Hayes